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SOME REMARKS ON THE LUBIN-TATE EXTENSIONS

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In this paper, we consider the possibility of characterization of the Lubin-Tate extensions, among the totally ramified abelian extensions over a local number field K, by means of their Galois groups.

The tamely ramified case is well known (Remark 1). In other cases, if K is a finite unramified local number field, the Lubin-Tate extension is characterized by the order and the exponent of its Galois group (Theorem). However, in general such characterization of Lubin-Tate extension is impossible; namely, we can find fields K over which their exist always other totally ramified abelian extensions whose Galois groups are isomorphic to those of Lubin-Tate extensions (Proposition 1).

Finally, we give a remark on the composite of two Lubin-Tate extensions (Proposition 2).

NOTATIONS. Z: the ring of rational integers. p: a prime number. Z_p: the ring of p-adic integers. Q_p: the field of p-adic numbers. K: a finite extension of Q_p. π: a prime element of K. $\mathfrak{p}$: the maximal ideal of K. U: the group of units of K. H_m: the multiplicative group $1+\mathfrak{p}^m$ ($m=1, 2, \cdots$). q: the number of elements of the residue class field of K. ρ: a primitive $(q-1)$-th root of unity in K. $M^\times$: the multiplicative group of a field M. $\langle\alpha\rangle$: the cyclic group generated by α. $N_{M/N}$: the norm map of a field extension M/N. $\mathrm{Gal}(M/N)$: the Galois group of a Galois extension M/N.

Now, the Lubin-Tate extension $L(\pi, m)$ is defined as follows; For $f(X)=X^q+\pi X$ let $\lambda_n(n=1, 2, \cdots)$ be elements of an algebraic closure of Q_p such that $f(\lambda_1)=0$ $(\lambda_1\neq 0)$, $f(\lambda_n)=\lambda_{n-1}(n\geqq 2)$ and we set $L(\pi, m)=K(\lambda_m)$.

Then $L(\pi, m)$ is a totally ramified abelian extension of K such that $N_{L(\pi,m)/K}(L(\pi, m)^\times)=\langle\pi\rangle H_m$ and $\mathrm{Gal}(L(\pi, m))\cong U/H_m$ (J. Lubin and J. Tate [3]).

THEOREM. *Let K/Q_p $(p\neq 2)$ be a finite unramified extension and M/K be a finite totally ramified abelian extension. Then $M\subseteqq L(\pi, m)$ for some π if and only if the exponent of* $\mathrm{Gal}(M/K)$ *is a divisor of $(q-1)p^{m-1}$. Moreover, if the order of* $\mathrm{Gal}(M/K)$ *is $(q-1)q^{m-1}$ then $M=L(\pi, m)$ for some π.*

Proof. "If" part: Let $N_{M/K}(U_M)=U'$ where U_M is the group of units of M. By class field theory $\mathrm{Gal}(M/K)\cong U/U'$. Since the exponent of $\mathrm{Gal}(M/K)$ is a

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