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MINIMAL HYPERSURFACES WITH THREE PRINCIPAL CURVATURE FIELDS IN S^{n+1}

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As is well known, there are many works on minimal hypersurfaces with two regular principal curvature fields in the space forms, especially the spheres. On the contrary, it seems we have very few works on minimal hypersurfaces with more than two principal curvature fields in such spaces. In the present paper, we shall study minimal hypersurfaces in S^{n+1} which have three regular and nonsimple principal curvature fields, say μ_1, μ_2 and μ_3.

In § 1, we shall state some fundamental theorems in the following argument. In § 2 and § 3, we shall develope a general theory on such hypersurfaces and find that the three tangent vector fields $H(\mu_i)$ (defined by (3.13)) corresponding to μ_i, $i=1, 2, 3$, play important role in our investigation. In § 4 and § 5, we shall treat the case in which one of $H(\mu_i)$ vanishes identically and give an example of such hypersurfaces. Finally, we shall investigate the case in which $H(\mu_i) \not\equiv 0$, $i=1, 2, 3$, and show that each μ_i can not be constant (Theorem 5), which tells us that in order to construct examples of such hypersurfaces each μ_i must be considered as a nonconstant function.

§ 1. Preliminaries

Let $M=M^n$ be a hypersurface in an $(n+1)$-dimensional Riemannian manifold $\bar{M}=\bar{M}^{n+1}$ of constant curvature $\bar{c}$. Let $\bar{\omega}_A$, $\bar{\omega}_{AB}=-\bar{\omega}_{BA}$, $A, B=1, 2, \cdots, n+1$, be the basic and connection forms of $\bar{M}$ on the orthonormal frame bundle $F(\bar{M})$ over $\bar{M}$, which satisfy the structure equations

$$(1.1) \qquad d\bar{\omega}_A=\sum_B \bar{\omega}_{AB}\wedge\bar{\omega}_B\,, \qquad d\bar{\omega}_{AB}=\sum_C \bar{\omega}_{AC}\wedge\bar{\omega}_{CB}-\bar{c}\bar{\omega}_A\wedge\bar{\omega}_B\,.$$

Let B be the submanifold of $F(\bar{M})$ over M composed of $b=(x, e_1, \cdots, e_{n+1})$ such that $(x, e_1, \cdots, e_n)\in F(M)$, where $F(M)$ is the orthonormal frame bundle of M with the induced Riemannian metric from $\bar{M}$. Then, deleting the bars of $\bar{\omega}_A$, $\bar{\omega}_{AB}$ on B, we have

$$(1.2) \qquad \omega_{n+1}=0\,, \qquad \omega_{i(n+1)}=\sum_j A_{ij}\omega_j\,, \qquad A_{ij}=A_{ji}\,,$$
$$i, j=1, 2, \cdots, n\,.$$

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