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UNICITY THEOREMS FOR ENTIRE FUNCTIONS CONCERNING FOUR SMALL FUNCTIONS

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Abstract

This paper studies the problem of uniqueness of entire functions concerning four small functions and shows that if two entire functions f and g satisfy $\bar{E}(a_j, k, f) = \bar{E}(a_j, k, g)$ for $j = 1, 2, 3, 4$, where a_j are four distinct small functions with respect to f and g, and k is a positive integer or ∞ with $k \geq 8$, then $f \equiv g$.

1. Introduction and main result

In this paper, by meromorphic function we shall always mean a meromorphic function in the complex plane $\boldsymbol{C}$. We adopt the standard notations in the Nevanlinna theory of meromorphic functions as explained in [1]. For any nonconstant meromorphic function $f(z)$, we denote by $S(r, f)$ any quantity satisfying $S(r, f) = o(T(r, f))$ as $r \to \infty$ except possibly for a set of r of finite linear measure. A meromorphic function $a(z)$ is called a small function with respect to $f(z)$ if $T(r, a) = S(r, f)$. Let $S(f)$ be the set of meromorphic functions in the complex plane $\boldsymbol{C}$ which are small functions with respect to f. Note that $\boldsymbol{C} \in S(f)$ and $S(f)$ is a field (see [2]).

Let $f(z)$ be a nonconstant entire function, $a(z) \in S(f)$, and let k be a positive integer or ∞. We denote by $\bar{E}(a, k, f)$ the set of distinct zeros of $f(z) - a(z)$ with multiplicities $\leq k$ (see [3]). In particular, we denote by $\bar{E}(a, \infty, f)$ the set of distinct zeros of $f(z) - a(z)$.

Let $f(z)$ and $g(z)$ be nonconstant entire functions and let $a(z) \in S(f) \cap S(g)$. We denote by $\bar{N}_0(r, a, f, g)$ the counting function of common zeros of $f(z) - a(z) = 0$ and $g(z) - a(z) = 0$ (ignoring multiplicities), each point counted only once. Let

$$\bar{N}_{12}(r, a, f, g) := \bar{N}(r, a, f) + \bar{N}(r, a, g) - 2\bar{N}_0(r, a, f, g), \tag{1.1}$$

then $\bar{N}_{12}(r, a, f, g)$ denotes the counting function of different solutions to $f(z) - a(z) = 0$ and $g(z) - a(z) = 0$. Set

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