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INTEGRAL FORMULAS AND THEIR APPLICATIONS IN QUATERNIONIC KÄHLERIAN MANIFOLDS

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Recently, quaternionic Kählerian manifolds have been studied by several authors (Alekseevskii [1], [2], Gray [3], Ishihara [4], [5], Ishihara and Konishi [6], Krainse [7] and Wolf [11]). On the other hand, Yano [8], [9] and Yano and Bochner [10] established some integral formulas in compact Kählerian manifolds and, using these integral formulas, obtained interesting results concerning Killing and analytic vectors in compact Kählerian manifolds. In the present note, we establish some integral formulas in compact quaternionic Kählerian manifolds and, using these integral formulas, prove some theorems concerning Killing vectors and vector fields preserving the quaternionic structure, which will be called infinitesimal Q-transformations.

In § 1, we recall definitions and some properties of quaternionic Kählerian manifolds. In § 2, we define infinitesimal Q-transformations in quaternionic Kählerian manifolds and give some properties of infinitesimal Q-transformations. § 3 is devoted to establish some integral formulas in compact quaternionic Kählerian manifolds for later use. In § 4, using integral formulas established in § 3, prove some theorems concerning Killing vectors and infinitesimal Q-transformations in compact quaternionic Kählerian manifolds.

Manifolds, mappings, tensor fields and other geometric objects we discuss are assumed to be differentiable and of class C^∞. The indices $h, \imath, \jmath, k, l, r, s, t$ run over the range $\{1, 2, \cdots, n\}$, $(n=4m,\ m\geqq 1)$ and the summation convension will be used with respect to this system of indices.

§ 1. Quaternion Kählerian manifolds.

Let M be a differentiable manifold of dimension n and assume that there is a subbundle V of the tensor bundle of type (1, 1) over M such that V satisfies the following condition:

(a) In any coordinate neighborhood U of M, there is a local basis $\{F, G, H\}$ of the bundle V, where F, G and H are tensor fields of type (1, 1) in U, and satisfy

$$\begin{aligned} &F^2=-I, && G^2=-I, && H^2=-I, \\ &GH=-HG=F, && HF=-FH=G, && FG=-GF=H, \end{aligned} \tag{1.1}$$

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