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ON CONFORMAL RIGIDITY OF A RIEMANN SURFACE

Dedicated to Professor Yûsaku Komatu on his 60th birthday

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1. A Riemann surface is said to be conformally rigid if the preservation of a certain condition attached to the Riemann surface implies that an analytic self-mapping is an automorphism. The notion of the homotopical or the homological (conformal) rigidity was treated by Huber [2], Landau and Osserman [4], Marden, Richards and Rodin [5] and Jenkins and Suita [3]. Jenkins and Suita have made clear the relationship between these two types of rigidity.

In this paper we shall give a criterion for a Riemann surface to be homotopically rigid. Furthermore, we shall introduce the notion of the weakly homological rigidity in a similar manner and shall treat the relationship among these three types of rigidity.

2. A Riemann surface W is said to be homotopically (resp. homologically or weakly homologically) rigid if every analytic self-mapping of W, which preserves the homotopical (resp. homological or weakly homological) non-triviality, reduces to an automorphism of W. Let $\mathfrak{S}$ denote the class of Riemann surfaces every non-constant analytic self-mapping of which reduces to a univalent mapping. Then we have

THEOREM 1. *If W is of $O_{HD}\cap\mathfrak{S}$, then it is homotopically rigid. $O_{HD}\cap\mathfrak{S}$ cannot be replaced with $\mathfrak{S}$.*

Heins [1] proved

THEOREM A. *Every Riemann surface of class O_G and having the non-abelian fundamental group is of class $\mathfrak{S}$.*

Therefore, we have

THEOREM 2. *If W is of class O_G and has the non-abelian fundamental group, then it is homotopically rigid.*

Next we shall show a criterion to be weakly homologically rigid. That is

THEOREM 3. *If W is of positive finite genus but not a torus, then it is weakly*

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