A. EDIGARIAN AND W ZWONEK
KODAI MATH. J.
22 (1999), 305–312

PROPER HOLOMORPHIC MAPPINGS IN SOME CLASS OF UNBOUNDED DOMAINS

ARMEN EDIGARIAN AND WŁODZIMIERZ ZWONEK

Abstract

In the paper the problem of the form of proper holomorphic mappings between elementary Reinhardt domains of the rational type in C^2 is solved. The pluricomplex Green function with many poles is used in the solution of the problem. Additionally, we solve the holomorphic equivalence problem of elementary Reinhardt domains of the arbitrary type in C^n The results in the paper are generalizations of those from two papers of S. Shimizu.

Let D be a domain in C^n. Fix a finite set $\emptyset \neq P \subset D$ and a function $v : P \to (0, \infty)$. Denote by $\mathscr{K}_{D,P,v}$ the family of functions satisfying the following conditions:

$$u \in \mathrm{PSH}(D),\ u < 0 \text{ and for any } p \in P, u(z) - v(p)\log\|z-p\| \text{ is bounded above for } z \text{ near } p$$

(we allow a plurisubharmanic function to be identically $-\infty$). Let us define *the pluricomplex Green function with poles in P and the weights v* (see [Lel]) as follows:

$$g_D(P; v; w) := \sup\{u(w) : u \in \mathscr{K}_{D,P,v}\}, \quad w \in D.$$

If $v \equiv 1$, then we denote $g_D(P; w) := g_D(P; v; w)$. In the special case $P = \{p\}$, $v(p) = 1$ we denote $g_D(p; w) := g_D(P; v; w)$ – it is a pluricomplex Green function with logarithmic pole at p defined by M. Klimek (see [Kli]).

The following inequalities are easy to verify:

$$\min\{v(p)g_D(p; w) : p \in P\} \geq g_D(P; v; w) \geq \sum_{p \in P} v(p)g_D(p; w). \tag{1}$$

It is also well-known that $g_D(P; v; \cdot) \in \mathscr{K}_{D,P,v}$.

Put $V_0 := \{z = (z_1, \dots, z_n) \in C^n : z_1 \cdots z_n = 0\}$.

Let $\alpha = (\alpha_1, \dots, \alpha_n) \in (R_+)^n$, where $n > 1$ (R_+ stands for positive numbers). Define

$$D_\alpha := \{z = (z_1, \dots, z_n) \in C^n : |z_1|^{\alpha_1} \cdots |z_n|^{\alpha_n} < 1\}.$$

Research partially supported by the KBN grant No. 2 PO3A 060 08.
Received August 20, 1997; revised April 28, 1999.