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THE SPECTRAL GEOMETRY OF HARMONIC MAPS INTO $HP^n(c)$

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§0. Introduction

The spectral geometry of the Laplace-Beltrami operator has developed greatly during the last twenty years. Recently, H. Urakawa use Gilkey's results about the asymptotic expansion of the trace of the heat kernel of a certain differential operator of a vector bundle to research the spectral geometry of harmonic maps into S^n and CP^n. In this paper, inspired by these, we firstly determine a spectral invariant of the Jacobi operator of harmonic maps into HP^n (corollary 3). Using this we obtain some geometric results distinguishing typical harmonic maps, i.e., isometric minimal immersions and Riemannian submersions with minimal fibres.

§1. The spectral invariants of the Jacobi operator

Let (M, g) be a m-dimentional compact Riemmanian manifold without boundary and (N, h) an n-dimentional Riemannian manifold. A smooth map $\phi:(M, g)\to(N, h)$ is said to be harmonic if it is a critical point of the energy $E(\phi)$ defined by

$$E(\phi) = \int_M e(\phi)vg \tag{1}$$

$$e(\phi) = \frac{1}{2}\sum_{i=1}^{m} h(\phi_* e_i, \phi_* e_i) \tag{2}$$

where ϕ_* is the differential of ϕ. Namely, for every vector field V along ϕ

$$\left.\frac{d}{dt}\right|_{t=0} E(\phi_t) = 0.$$

Here $\phi_t: M\to N$ is a one parameter family of smooth maps with $\phi_0=\phi$ and

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