O. SAEKI
KODAI MATH. J.
17 (1994), 518—529

STABLE MAPS AND LINKS IN 3-MANIFOLDS

BY OSAMU SAEKI

1. Introduction

Let $h : S^3 \to \mathbf{R}$ be the standard Morse function with exactly two critical points. It is known that, if K is an embedded circle (or a knot) in S^3 such that $h|K : K \to \mathbf{R}$ is a Morse function with exactly two critical points, then K is trivial (i.e., it bounds an embedded 2-disk in S^3). The main purpose of this paper is to study links — finite disjoint union of embedded circles — in 3-manifolds using stable maps into 2-manifolds (or surfaces) instead of Morse functions. This is a continuation of the study begun in [**S1**,§6].

Let $g : M^3 \to N^2$ be a smooth map of a closed 3-manifold into a surface. Then g can be approximated, in the sense of C^∞-topology, by a stable map $f : M^3 \to N^2$, which can be regarded as a variant of Morse functions. Thus there are plenty of stable maps on a 3-manifold. The singularities of a stable map can be written down by normal forms explicitly, as non-degenerate critical points of a Morse function can be given by explicit normal forms by the Morse Lemma ([**Mi**]). In fact, there are exactly three types of singularities for a stable map: definite fold points, indefinite fold points, and cusp points. Stable maps have been studied by many authors [**L1, L2, BdR, KLP, ML1, ML2, ML3, ML4, S1, S2, S3, MPS**] and a lot of interesting results have been obtained.

Given a link L in M^3, we can always change L by an isotopy so that $f|L : L \to N^2$ is an immersion with normal crossings. In this paper we try to obtain information on L using f and the immersion $f|L$. In [**S1**] we have considered the case where f is a simple stable map, and have given a characterization of graph links in terms of such maps. In this paper, we consider a more restricted class of maps, namely full-definite simple stable maps ([**S2**]), and show that, if $f|L$ is an embedding whose image contains no critical value for a full-definite simple stable map $f : S^3 \to \mathbf{R}^2$ and a link L in S^3, then L is trivial (i.e., L bounds disjoint embedded 2-disks in S^3).

Another important fact about stable maps is that the singular point set $S(f)$ of a stable map $f : M^3 \to N^2$ is a smooth closed 1-dimensional submanifold of M^3; i.e., it is a link in M^3. Furthermore, the regular fiber $f^{-1}(a)$ for a regular value $a \in N^2$ is also a link in M^3. Note that for every link L in S^3, there exists a stable map $f_1 : S^3 \to \mathbf{R}^2$ whose singular set $S(f_1)$ coincides with L. There also exists a stable map $f_2 : S^3 \to \mathbf{R}^2$ such that $f_2^{-1}(a) = L$ for a regular value $a \in \mathbf{R}^2$. In this paper, using the above facts, we define integer invariants of a link L in S^3, which measure a kind of complexity of such maps as f_1 and f_2 above. Although these invariants are thus defined properly, we

The author is partially supported by CNPq, Brazil.
Received May 18, 1993.