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Sato's conjecture on recurrence conditions for multidimensional processes of Ornstein-Uhlenbeck type

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1. Introduction

A stochastic process of Ornstein-Uhlenbeck type (OU type process) $\{X_t\}$ was introduced in one dimension by Wolfe [7] and in multidimension by Sato and Yamazato [4]. It is a Markov process $(\Omega, \mathscr{F}, \mathscr{F}_t, P^x, X_t)$ on the d-dimensional Euclidean space $\boldsymbol{R}^d$ obtained from a spatially homogeneous Markov process undergoing a linear drift force determined by a matrix $-Q$. The purpose of this paper is to give an integral condition of recurrence and transience for OU type processes. Let $\{Z_t\}$ be a Lévy process on $\boldsymbol{R}^d$, that is, a stochastically continuous process with stationary independent increments, starting at the origin. Let Q be a real $d \times d$ matrix of which all eigenvalues have positive real parts. An OU type process $\{X_t\}$ on $\boldsymbol{R}^d$ is, under the measure P^x, equivalent to the process $\{\overline{X}_t\}$ defined by

$$\overline{X}_t = e^{-tQ}x + \int_0^t e^{-(t-u)Q}\, dZ_u, \tag{1.1}$$

where the stochastic integral with respect to the Lévy process $\{Z_t\}$ is defined by stochastic convergence from integrals of simple functions. It is the unique solution of the equation

$$\overline{X}_t = x + Z_t - \int_0^t Q\overline{X}_u\, du. \tag{1.2}$$

An OU type process is determined by the Lévy process $\{Z_t\}$ and the matrix Q. When $\{Z_t\}$ is a Brownian motion and Q is a positive constant multiple of the unit matrix, it is a classical Ornstein-Uhlenbeck process. Precise definition of an OU type process by its infinitesimal generator is given in [2] and [4]. The process $\{X_t\}$ is called recurrent if there is $y \in \boldsymbol{R}^d$ such that

$$P^x(\liminf_{t\to\infty} |X_t - y| = 0) = 1 \quad \text{for every } x \in \boldsymbol{R}^d.$$

The process $\{X_t\}$ is called transient if

$$P^x(\lim_{t\to\infty} |X_t| = \infty) = 1 \quad \text{for every } x \in \boldsymbol{R}^d.$$

OU type processes are necessarily recurrent if they have limit distributions. Sato and Yamazato [3, 4] obtain a necessary and sufficient condition for OU type processes to have limit distributions. Moreover they show in [4], by giving a concrete example, that