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Linear differential equations with rational coefficients

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1. Introduction and results.

We consider the nth order linear differential equation

$$w^{(n)}+r_{n-2}(z)w^{(n-2)}+\cdots+r_0(z)w=0 \tag{1}$$

with rational coefficients. Unlike the case of polynomial coefficients, we know, Eq. (1) may have some solutions multivalued in the complex plane. In this paper, however, we always assume that each solution of Eq. (1) is single-valued, meromorphic in the complex plane. Then it is clear to see that each solution of Eq. (1) has only a finite number of poles, so that its Valiron deficient at ∞ is equal to 1.

Now let us introduce some notations. For a system of rays

$$D=\bigcup_{j=1}^{m}\{z\,|\arg z=\theta_j\},\ 0\leqq\theta_1<\cdots<\theta_m<\theta_{m+1}=\theta_1+2\pi, \tag{2}$$

we define

$$\omega(D)=\max\left\{\frac{\pi}{\theta_{j+1}-\theta_j}\,|\,1\leqq j\leqq m\right\}$$

and

$$G(D,\varepsilon)=\boldsymbol{C}\setminus\bigcup_{j=1}^{m}\{z\,|\,|\arg z-\theta_j|<\varepsilon\}.$$

Let $f(z)$ be a function meromorphic in the complex plane. We shall say that the zeros of $f(z)$ are attracted to D, provided that for any $\varepsilon>0$

$$n\left(r,G(D,\varepsilon),\frac{1}{f}\right)=o(T(r,f)), \tag{3}$$

as $r\to+\infty$, where $n(r,G(D,\varepsilon),1/f)$ is the number of zeros of $f(z)$ lying in $G(D,\varepsilon)\cap\{|z|<r\}$. And we always denote the order and the lower order of $f(z)$ by $\lambda(f)$ and $\rho(f)$, respectively. We assume that the reader is familiar with Nevanlinna theory of meromorphic functions and standard notations.

THEOREM 1. *Assume that Eq.* (1) *is given with rational coefficients* $r_j(z)$. *Suppose that there exists a fundamental set* (*FS*) $\{w_1,\cdots,w_n\}$ *of Eq.* (1) *with the*