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$L_{n/2}$-pinching theorems for submanifolds with parallel mean curvature in a sphere

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1. Introduction.

Let M^n be an n-dimensional oriented closed minimal submanifold in the unit sphere $S^{n+p}(1)$. We denote the square of the length of the second fundamental form by S. It is well known [**10**] that if $S<n/(2-1/p)$ on M, then $S\equiv 0$ and hence M is isometric to the unit sphere $S^n(1)$. Further discussions in this direction have been carried out by many other authors ([**4**], [**7**], [**12**], etc.), but all these results have pointwise condition for S. It seems to be interesting to study the L_q-pinching condition for S. By using eigenvalue estimate, Shen [**9**] proved the following

THEOREM A. *Let* $M^n \to S^{n+1}(1)$ *be an oriented closed embedded minimal hypersurface with* $\mathrm{Ric}_M \geqq 0$. *If* $\int_M S^{n/2}<C_1'(n)$, *where* $C_1'(n)$ *is a positive universal constant, then* M *is a totally geodesic hypersurface.*

By using Gauss-Bonnet Theorem and a generalized Simons' inequality, Lin and Xia [**6**] proved the following

THEOREM B. *Let* M^{2m} *be an even dimensional oriented closed minimal submanifold in* $S^{2m+p}(1)$. *If the Euler characteristic of* M *is not greater than two, and* $\int_M S^m<C_2'(m, p)$, *where* $C_2'(m, p)$ *is a positive universal constant depending on* m *and* p, *then* M *is totally geodesic.*

In the present paper, we will study the $L_{n/2}$-pinching problem for n-dimensional compact submanifolds with parallel mean curvature in the unit sphere $S^{n+p}(1)$. Now we define our pinching constants as follows

$$\alpha(n, H)=\frac{2na(n)}{C^2(n)[(a(n)b(n, H))^{1/2}+(1+a(n))^{1/2}(2+b(n, H))^{1/2}]^2}, \tag{1.1}$$

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