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Some conformal properties of p-harmonic maps and a regularity for sphere-valued p-harmonic maps

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1. Introduction.

Let $u: M \to N$ be a smooth map between Riemannian manifolds and p a real number $1<p<\infty$. We call u a p-harmonic map if it is a critical point of the p-energy functional $\int_{\Omega}|du(x)|^p dv_g$ for every compact domain $\Omega \subset M$. Since the p-energy functional is a natural generalization of the energy functional ($p=2$) for a harmonic map, it is an important problem to study the difference between p-harmonic maps ($p \neq 2$) and harmonic maps. In this paper we shall focus our study on conformal properties of p-harmonic maps and the regularity for sphere-valued p-harmonic maps.

Our main results are as follows. In Section 3, we show that for $p' \neq p$, $p \neq \dim M$, any p'-harmonic map becomes a p-harmonic one by some conformal change of a given metric on M. We also discuss their stability under this conformal change. In Section 4, we investigate p-harmonic conformal maps and, in particular, show relations between the mean curvature vectors and p-tension fields of these maps. Based on this observation we prove that if $\dim M = p$ and $\dim M < \dim N$, then a conformal map u is p-harmonic if and only if $u(M)$ is a minimal submanifold in N (Corollary 4). If $\dim M = p$ and $\dim M > \dim N$, then the fibres of p-harmonic horizontal conformal maps are minimal submanifolds in M (Proposition 7).

In Section 5, we discuss the regularity for sphere-valued weakly p-harmonic maps which are not necessarily minimum. Helein [**11**] has shown that any weakly harmonic map from a two-dimensional surface into a sphere is smooth. Evans [**7**] generalized this to higher dimensions. We prove a regularity theorem similar to the one of Evans for weakly p-harmonic maps ($p \geqq 2$) into a sphere. Namely, if U is a smooth open subset in $\boldsymbol{R}^m$ and S^{n-1} is the unit sphere in $\boldsymbol{R}^n$, then a weakly p-harmonic map from U into S^{n-1} is locally Hölder continuous on $\Omega \setminus \mathcal{S}_u$ for some compact set $\mathcal{S}_u$ whose $(m-p)$-dimensional Hausdorff measure is 0. In particular, in the case $m=p$, p-harmonic maps are