J. Math. Soc. Japan
Vol. 45, No. 2, 1993

Application of the theory of KM_2O-Langevin equations to the linear prediction problem for the multi-dimensional weakly stationary time series

Dedicated to Professor Hiroshi Tanaka on his sixtieth birthday

By Yasunori OKABE

(Received Feb. 7, 1992)

§ 1. Introduction.

Let $X=(X(n); n\in\mathbb{Z})$ be a d-dimensional weakly stationary time series on a probability space $(\Omega, \mathcal{B}, P)$ with expectation vector 0 and covariance matrix function R. Wiener and Masani ([16], [17], [3]) have developed a theory of the linear prediction problem for the time series X. By the innovation method, they have introduced an innovation process $\boldsymbol{\varepsilon}_+=(\varepsilon_+(n); n\in\mathbb{Z})$ by

$$\varepsilon_+(n)=X(n)-P_{M_{-\infty}^{n-1}(X)}X(n), \tag{1.1}$$

where $P_{M_{-\infty}^{n-1}(X)}$ stands for the projection operator on the past subspace $\boldsymbol{M}_{-\infty}^{n-1}(\boldsymbol{X})$ of $L^2(\Omega, \mathcal{B}, P)$ defined by

$$\boldsymbol{M}_{-\infty}^{n-1}(\boldsymbol{X})=\text{the closed subspace generated by } \{X_j(l); l\leqq n-1, 1\leqq j\leqq d\}. \tag{1.2}$$

We denote by $V_+\in M(d; \mathbb{R})$ the prediction error matrix:

$$V_+=E(\varepsilon_+(0)^t\varepsilon_+(0)). \tag{1.3}$$

The process $\boldsymbol{X}$ is said to be purely nondeterministic if and only if $\bigcap_{n\in\mathbb{Z}}\boldsymbol{M}_{-\infty}^{n}(\boldsymbol{X})=0$ and to be of full rank if and only if the rank of the matrix V_+ is d. It has been in [16] characterized that $\boldsymbol{X}$ is purely nondeterministic and of full rank if and only if it has a spectral density matrix function Δ such that

$$\log(\det(\Delta))\in L^1(-\pi, \pi) \tag{1.4}$$

and then proved that

$$\det V_+=\exp\left[\frac{1}{2\pi}\int_{-\pi}^{\pi}\log(\det(\Delta(\theta)))d\theta\right]. \tag{1.5}$$

This research was partially supported by Grant-in-Aid for Science Research No. 03452011, the Ministry of Education, Science and Culture, Japan.