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On affine hypersurfaces with parallel nullity

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Affine differential geometry for hypersurfaces in the classical sense of Blaschke is based on the hypothesis that the given hypersurface is nondegenerate; quote from [**B**, p. 104]: Für parabolisch gekrümmte Flächen ("Torsen", $LN-M^2=0$) versagt die Grundform. In relative geometry (for example, see [**S**]) and in the study of affine immersions [**N-P1**], [**N-P2**], the nondegeneracy condition is often important, although a few results (for example, Berwald's theorem [**N-P2**], Radon's theorem [**O**]) have been established under a somewhat weaker assumption on the rank of the fundamental form h.

In this paper, we examine a general condition weaker than nondegeneracy under which geometry of a given hypersurface can be reduced to the classical situation. We start with an immersion $f: M^n \to R^{n+1}$. *For an arbitrary choice of a transversal vector field* ξ, *consider the condition that the kernel* T^0 *of* h *be parallel relative to the connection* ∇ *induced by* ξ. *It turns out that this condition is independent of the choice of* ξ. Under this condition of parallel nullity and under a completeness assumption which is also intrinsic, we shall show that f is globally a cylinder immersion of the form $M^n = M^r \times L$, $f = f_1 \times f_0$, where $f_1: M^r \to R^{r+1}$ is a nondegenerate hypersurface, L is a leaf of T^0, and f_0 is a connection-preserving map of L onto R^{n-r}, where R^{r+1} and R^{n-r} are affine subspaces in R^{n+1} that are mutually transversal. Such a representation is unique up to equiaffine transformation. Thus the geometry of M^n is completely determined by that of a profile nondegenerate hypersurface M^r in R^{r+1} that is itself uniquely determined up to equiaffine equivalence. For later applications we include additional information on transversal vector fields.

1. Preliminaries.

Let $f: M^n \to R^{n+1}$ be a connected hypersurface immersed in the affine space R^{n+1} provided with a fixed determinant function (volume element). Around

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