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On the Gauss curvature of minimal surfaces

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§ 1. Introduction.

In 1952, E. Heinz showed that, for a minimal surface M in $\boldsymbol{R}^3$ which is the graph of a function $z=z(x, y)$ of class C^2 defined on a disk $\Delta_R := \{(x, y); x^2+y^2<R^2\}$, there is a positive constant C not depending on each surface M such that $|K| \leqq C/R^2$ holds for the curvature K of M at the origin ([8]). This is an improvement of the classical Bernstein's theorem that a minimal surface in $\boldsymbol{R}^3$ which is the graph of a function of class C^2 defined on the total plane is necessarily a plane. Later, R. Osserman gave some generalizations of these results to surfaces which need not be of the form $z=z(x, y)$ ([**10**], [**11**]). To state one of his results, we consider a connected, oriented minimal surface M immersed in $\boldsymbol{R}^3$ and, for a point $p \in M$, we denote by $K(p)$ and $d(p)$ the Gauss curvature of M at p and the distance from p to the boundary of M respectively. He gave the following estimate of the Gauss curvature of M.

THEOREM A. *Let M be a simply-connected minimal surface immersed in $\boldsymbol{R}^3$ and assume that there is some fixed nonzero vector n_0 and a number $\theta_0>0$ such that all normals to M make angles of at least θ_0 with n_0. Then,*

$$|K(p)|^{1/2} \leqq \frac{1}{d(p)} \frac{2\cos(\theta_0/2)}{\sin^3(\theta_0/2)} \qquad (p \in M).$$

He obtained also some generalization of Theorem A to minimal surfaces immersed in $\boldsymbol{R}^m\,(m \geqq 3)$ ([**12**]).

Relating to these results, the author proved the following theorem in his paper [**4**].

THEOREM B. *Let M be a minimal surface immersed in $\boldsymbol{R}^3$ and let $G: M \to S^2$ be the Gauss map of M. If G omits mutually distinct five points $n_1, \cdots, n_5$ in S^2, then it holds that*

$$(1) \qquad |K(p)|^{1/2} \leqq \frac{C}{d(p)} \qquad (p \in M)$$

for some positive constant C depending only on n_j's.