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Interpolating sequences in the maximal ideal space of H^∞

By Keiji IZUCHI

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1. Introduction.

Let H^∞ be the space of bounded analytic functions on the open unit disc D. H^∞ becomes a Banach algebra with the supremum norm. We denote by $M(H^\infty)$ the maximal ideal space of H^∞ with the weak*-topology. We identify a function in H^∞ with its Gelfand transform. For points x and y in $M(H^\infty)$, the pseudo-hyperbolic distance is defined by

$$\rho(x, y)=\sup\{|h(x)|\,;\ h\in \text{ball }(H^\infty),\ h(y)=0\},$$

where ball (H^∞) stands for the unit closed ball of H^∞. For z and w in D, we have $\rho(z, w)=|z-w|/|1-\bar{z}w|$. A sequence $\{x_j\}_j$ in $M(H^\infty)$ is called interpolating if for every bounded sequence $\{a_j\}_j$ there is a function f in H^∞ such that $f(x_j)=a_j$ for every j. It is well known (see [2, p. 283]) that for a sequence $\{z_j\}_j$ in D, $\{z_j\}_j$ is interpolating if and only if

$$\inf_k \prod_{j\neq k}\rho(z_j, z_k)>0.$$

For a sequence $\{z_j\}_j$ in D with $\sum_{j=1}^\infty 1-|z_j|<\infty$, a function

$$b(z)=\prod_{j=1}^\infty \frac{\bar{z}_j}{|z_j|}\frac{z_j-z}{1-\bar{z}_j z}\quad (z\in D)$$

is called a Blaschke product with zeros $\{z_j\}_j$, and $\{z_j\}_j$ is called the zero sequence of b. If $\{z_j\}_j$ is interpolating, we call b interpolating. For a function f in H^∞, put $Z(f)=\{x\in M(H^\infty);\ f(x)=0\}$. For a subset E of $M(H^\infty)$, we denote by $\mathrm{cl}\,E$ the weak*-closure of E in $M(H^\infty)$.

For a point x in $M(H^\infty)$, the set $P(x)=\{y\in M(H^\infty);\ \rho(y, x)<1\}$ is called a Gleason part of x. If $P(x)\neq\{x\}$, $P(x)$ is called nontrivial. D is a typical nontrivial part. We set

$$G=\{x\in M(H^\infty);\ x \text{ is nontrivial}\}.$$

Hoffman [5] proved that for a point x in G, there is an interpolating sequence $\{z_j\}_j$ such that x is contained in $\mathrm{cl}\,\{z_j\}_j$, and there is a continuous map L_x from D onto $P(x)$ such that $f\circ L_x\in H^\infty$ for every $f\in H^\infty$, where L_x is given