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Differentiable sphere theorem by curvature pinching

Dedicated to Professor Nobuyuki Ikeda on his 60th birthday

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§ 0. Introduction.

An important problem in differential geometry is to characterize the global behaviour of a manifold in terms of local invariants. A result in this direction is given by the following theorem: If M is a complete, simply connected riemannian manifold whose curvature tensor is close to the curvature tensor of the standard sphere S, then M is diffeomorphic to S. This is called the differentiable sphere theorem. In this paper, we prove that 0.681-pinched riemannian manifold is diffeomorphic to the standard sphere.

The proximity of curvature tensors R and $\bar{R}$ of the manifold M and the standard sphere S respectively is measured in terms of sectional curvature: A riemannian manifold whose sectional curvature K satisfies the condition $\delta \leqq K \leqq 1$ is called δ-pinched. For the first time, Gromoll [2], Calabi, and Shikata [**11**] gave some results on the differentiable sphere theorem. Later on, these results were improved: Sugimoto and Shiohama [**12**] found a pinching number $\delta(=0.87)$ independent of the dimension of M such that a complete, simply connected and δ-pinched riemannian manifold M is diffeomorphic to the standard sphere. Im Hof and Ruh [**5**] gave a sequence δ_n of pinching numbers dependent on n of dimension of M: A δ_n-pinched manifold M is not only diffeomorphic to the standard sphere, but the action of the isometry group of M is also equivalent to the standard linear action of a subgroup of $O(n+1, \boldsymbol{R})$ on the sphere. The number δ_n is decreasing on n and $\lim \delta_n = 0.68$ as n tends to infinity. But, if we take the number δ independent of dimension of M on Im Hof and Ruh's result, δ becomes considerably large, i.e., $\delta=0.98$ for $n>5$. It is unknown what number is the infimum of δ in order that a complete, simply connected and δ-pinched riemannian manifold is diffeomorphic to the standard sphere.

Sugimoto and Shiohama's beginning idea was due to Omori [7], from which they derived that a complete, simply connected and δ-pinched riemannian manifold M^n is diffeomorphic to the standard sphere S^n if a diffeomorphism f of S^{n-1}, which is naturally defined for δ-pinched manifold M, is diffeotopic to the