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An elementary proof of Yoshida's inequality for block designs which admit automorphism groups

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1. Introduction.

The main purpose of this paper is to give an elementary proof of Yoshida's inequality [5]. An incidence structure is a triple $D=(X, B, \mathcal{I})$, where X is a set of points, B is a set of blocks and $\mathcal{I}$ is a relation of incidence between points and blocks. A 2-(v, k, λ) design is an incidence structure $(X, B, \mathcal{I})$ satisfying the following requirements:

(1) $|X|=v$.
(2) Each block is incident with k points.
(3) Any 2 points are incident with λ blocks.

A 2-(v, k, λ) design is often called a block design. Let b be the total number of blocks. Note that each point of X is contained in exactly r blocks. We set $n=r-\lambda$, and we call n the order of the 2-design $(X, B, \mathcal{I})$. These parameters satisfy the following relations:

$$vr = bk\,, \qquad (v-1)\lambda = r(k-1)\,. \tag{1}$$

The incidence matrix A of a block design $(X, B, \mathcal{I})$ is the $v\times b$ matrix whose rows are indexed by points and whose columns are indexed by blocks, with the entry in row x and column β being 1 if $x\mathcal{I}\beta$ and 0 otherwise. (The notation "$x\mathcal{I}\beta$" means that x is incident with β.) The conditions that $(X, B, \mathcal{I})$ is a block design can be expressed in terms of A:

$$AJ = rJ\,, \qquad JA = kJ\,, \tag{2}$$
$$AA^t = nI+\lambda J\,. \tag{3}$$

(Here, throughout this paper, I is the identity matrix and J the matrix with every entry 1 of appropriate size.) From (3) it follows that if $\lambda<r$, then

$$\det(AA^t) = rkn^{v-1} \neq 0\,,$$

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