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On Siegel domains of finite type

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Introduction.

In this paper we introduce a new class of homogeneous Siegel domains, called Siegel domains of finite type. Let $D=D(V, F)\subset \boldsymbol{C}^N$ be a Siegel domain associated with a convex cone V and a V-hermitian form F. Let G_h (resp. G_a) be the identity component of the holomorphic (resp. affine) automorphism group of D. It is known (Nakajima [7]) that D is G_h-equivariantly and holomorphically imbedded, together with the ambient space $\boldsymbol{C}^N$, into a complex coset space M of the complexification of G_h. D is said to be *of finite type*, if there are only finitely many G_h-orbits in M. This concept is realization-free and is determined only by the holomorphic equivalence class of D. Let H be the identity component of the linear automorphism group of D. Then there exists a natural homomorphism ρ of H into the linear automorphism group of the cone V. The cone V is called of *$\rho(H)$-finite type*, if there exists only a finite number of $\rho(H)$-orbits in the ambient vector space in which V is imbedded as an open cone.

The first aim of this paper is to show that, if M has at most countably many G_h-orbits, then D is of finite type, and in this case each G_h-orbit is a semi-analytic set in M (Theorem 3.8). It follows that, if D is of finite type, then it is necessarily homogeneous (Proposition 3.11). As a consequence, if D is not homogeneous, then M has non-countably many G_h-orbits (Corollary 3.12). The main purpose of this paper is to prove the equivalence between finite type for D and $\rho(H)$-finite type for V (cf. Theorem 3.15). Thus D being of finite type or not is reduced to the problem on orbits under a group of linear transformations. We also show that every connected component of the intersection of a G_h-orbit with $\boldsymbol{C}^N$ is a G_a-orbit and conversely every G_a-orbit is obtained in this manner (Theorem 3.3). This yields a qualitative proof of a result of Kaup-Matsushima-Ochiai [5] which states that, if D is homogeneous, then it is affinely homogeneous (Corollary 3.5). Finally we remark that the class of Siegel domains of finite type properly contains the class of quasi-symmetric Siegel domains (Corollary 3.14 and Example 3.17). In this paper we make use of some

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