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The Euler characteristics and Weyl's curvature invariants of submanifolds in spheres

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§1. Introduction.

Let S^n be a unit n-sphere in R^{n+1}. Let M^p be a compact orientable p-dimensional Riemannian manifold which is imbedded in S^n. Let $\chi(M^p)$ be the Euler characteristics of M^p and $\tau(M^p)$ be the total curvature of M^p. One of Teufel's main results in [8] can be stated as follows.

$$\chi(M^p)=\tau(M^p)+\frac{1}{C_{n-1,n+1}}\int_{G_{n-1,n+1}}\chi(M^p\cap R^{n-1})\Omega_{n-1,n+1} \qquad \text{for } 3\leqq p, \tag{1.1}$$

where $G_{n-1,n+1}$ is the oriented Grassmann manifold of all oriented $(n-1)$-dimensional linear subspaces of R^{n+1}, $C_{n-1,n+1}$ its volume and $\Omega_{n-1,n+1}$ its standard volume element. Denote by $V(M^p)$ the volume of M^p. We can show (Theorem 4 in §4)

$$\chi(M^2)=\tau(M^2)+\frac{1}{2\pi}V(M^2). \tag{1.2}$$

In 1939, Weyl [10] found the formula for the volume of a tube of radius r about M^p. The coefficients in the power series expansion of the volume are expressed by the curvature invariants $k_e(M^p)$ (e even, $0\leqq e\leqq p$) (see (2.1)), which depend on the intrinsic geometry of M^p. Notice that $k_0(M^p)=V(M^p)$. Let $\tau_e(M^p)$ $(1\leqq e\leqq p)$ be the e-th total mean curvature of M^p (see (2.2)). Then we have $\tau(M^p)=\tau_p(M^p)$, $\tau_e(M^p)=0$ for e odd, and for e even

$$\tau_e(M^p)=\frac{(p-2)!!}{(2\pi)^{p/2}(n-p+e-2)!!\binom{p}{e}}k_e(M^p), \tag{1.3}$$

where we mean that $m!!=m(m-2)\cdots 4\cdot 2$ or $m!!=m(m-2)\cdots 3\cdot 1$ according as m is even or odd. S. S. Chern [2] gives the kinematic formula and the linear kinematic formula in R^n. Following Chern, we introduce curvature invariants

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