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Local Lie algebra structure and momentum mapping

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§ 1. Introduction.

Momentum mappings for symplectic actions on a symplectic manifold are group theoretical analogues of the linear and angular momentum associated with the translational and rotational invariance. The existence of (coadjoint equivariant) momentum mappings is important in the mechanics because they give some conservative quantities. This result is known as Noether's theorem (cf. [**6**, **9**]). There are some works which discuss whether a given symplectic action admits a (coadjoint equivariant) momentum mapping or not (cf. [**6**, **9**]).

A Poisson manifold M is a differentiable manifold with a Lie algebra structure on $C^{\infty}(M)$ which is a derivation in each of its arguments. So, Poisson manifolds are a generalization of symplectic manifolds. A Poisson bracket $\{ , \}$ on M is one-to-one corresponding to an exterior contravariant 2-tensor field P on M satisfying the Schouten bracket $[P, P]=0$ (P is called a Poisson tensor on M) by the following relation $\{f, g\}=\langle P \mid df \wedge dg\rangle=-[[P, f], g]$ (cf. [**3**, **5**, **7**]). Weinstein [**10**] studied the local structure of general Poisson structures and says that every Poisson manifold is essentially a union of symplectic manifolds which fit together in a smooth way. But a general Poisson structure is quite different from the Poisson structure induced from a symplectic structure in some aspects. For example, the center of Poisson algebra $C^{\infty}(M)$ induced from the symplectic structure of M is the 0-dimensional de Rham cohomology group $H^{0}(M, \boldsymbol{R})$. The center of Poisson algebra $C^{\infty}(M)$ of the general Poisson manifold M (functions in the center of the Poisson algebra $C^{\infty}(M)$ are called Casimir functions on M) is not so obvious.

We can consider momentum mappings of natural actions on Poisson manifolds (we will call these Poisson actions) through analogy with symplectic actions on symplectic manifolds. Since Noether's theorem for momentum mappings of a Poisson action holds good (cf. [**3**, **5**]), the notion of momentum mappings is also important in Poisson manifolds with symmetry, so it is interesting to study the existence or coadjoint equivariancy of momentum mappings

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