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A theorem on $P_\kappa(\lambda)$

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1. Introduction.

Let κ be a regular uncountable cardinal. Consider the set

$$S(\kappa, \kappa^+) = \{x \in P_\kappa(\kappa^+) : |x| = |x \cap \kappa|^+\} .$$

A question has been raised, notably in [1], [2] and [5], whether $S(\kappa, \kappa^+)$ can be stationary.

By a result of Baumgartner, cf. [1], if $S(\kappa, \kappa^+)$ is stationary then κ is (weakly) inaccessible and $0^\#$ exists. We show (in Corollary 4.3) that if $S(\kappa, \kappa^+)$ is stationary then the function $f(\xi) = \xi^+$ on κ has the Galvin-Hajnal norm κ^+.

2. Some facts about $P_\kappa(\lambda)$.

Throughout this paper, κ is a fixed regular uncountable cardinal; all other greek letters denote ordinal numbers. If x is a set of ordinals, then

$$\bar{x} = \text{the order type of } x ;$$

as usual, $|x|$ is the cardinality of x. For $\lambda \geq \kappa$,

$$P_\kappa(\lambda) = \{x \subset \lambda : |x| < \kappa\}.$$

2.1. DEFINITION ([4]). A set $C \subseteq P_\kappa(\lambda)$ is *closed* if whenever $D \subseteq C$ is a chain under inclusion with $|D| < \kappa$, then $\bigcup D \in C$. C is *unbounded* if for every $x \in P_\kappa(\lambda)$ there is a $y \in C$ with $x \subseteq y$. C is a *club* if it is closed and unbounded. A set $S \subseteq P_\kappa(\lambda)$ is *stationary* if $S \cap C \neq \emptyset$ for all clubs C.

2.2. PROPOSITION. *A subset C of κ is a club iff C is a club in $P_\kappa(\kappa)$; also, κ is a club in $P_\kappa(\kappa)$.*

2.3. PROPOSITION. *Let $\kappa \leq \alpha \leq \beta$. If C is a club in $P_\kappa(\alpha)$ then the set*

$$\{x \in P_\kappa(\beta) : x \cap \alpha \in C\}$$

is a club in $P_\kappa(\beta)$.

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