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On pluricanonical maps for 3-folds of general type

By Kenji MATSUKI

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§ 0. Introduction.

Throughout this paper, we fix the complex number field $\boldsymbol{C}$ as the ground field. The purpose of this paper is to prove the following

MAIN THEOREM. *Let X be a nonsingular projective 3-fold whose canonical divisor K_X is nef and big* (cf. M. Reid [**12**] or § 1). *Then*
(i) *$\Phi_{|7K_X|}$ is birational with the possible exceptions of*
   a) *$\chi(\mathcal{O}_X)=0$ and $K_X^3=2$, or*
   b) *$|3K_X|$ is composed of pencils, i.e.,* $\dim \Phi_{|3K_X|}(X)=1$,
(ii) *$\Phi_{|nK_X|}$ is birational for $n \geqq 8$. Further if $\chi(\mathcal{O}_X)<0$, e.g. when K_X is ample, $\Phi_{|nK_X|}$ is birational for $n \geqq 7$.*

X. Benveniste [**1**] proved that $\Phi_{|nK_X|}$ is birational for $n \geqq 9$ under the same assumption as ours. Our proof follows mainly his ideas but improves the result to the extent that it guarantees $\Phi_{|nK_X|}$ being birational for $n \geqq 7$ if $\chi(\mathcal{O}_X)<0$.

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§ 1. Preliminaries.

Let X be a nonsingular complete variety, and $D \in \mathrm{Div}(X) \otimes \boldsymbol{Q}$, where $\mathrm{Div}(X)$ is a free abelian group generated by Weil divisors on X. Then D is called nef if $D \cdot C \geqq 0$ for any curve C on X, and big if $\kappa(D, X)=\dim X$ (cf. Iitaka [**6**]), respectively. We denote the linear equivalence and the numerical equivalance by $\sim$ and $\approx$, respectively. For $D \in \mathrm{Div}(X)$ with $h^0(X, \mathcal{O}_X(D)) \neq 0$, $\Phi_{|D|}$ denotes the rational map associated with the complete linear system $|D|$.

PROPOSITION 1. *Let X be a nonsingular complete variety, and $D \in \mathrm{Div}(X) \otimes \boldsymbol{Q}$. Assume the following two conditions:*
(i) *D is nef and big,*
(ii) *the fractional part of D has the support with only normal crossings.*
Then

$$H^i(X, \mathcal{O}_X(\ulcorner D \urcorner + K_X))=0 \qquad \text{for } i>0,$$