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Remarks on the fixed point algebras of product type actions on UHF-algebras

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1. Introduction.

In this note we consider a C^*-dynamical system (A, G, α) of product type action, where A is a UHF-algebra and G is a finite group. In [**5**] and [**6**], A. Kishimoto and N. J. Munch investigated properties of the C^*-dynamical system (A, G, α). One of their results is that if G is abelian, then the space of tracial states on the fixed point algebra A^G is n-simplex where the number n is the cardinality of the subgroup of G which is weakly inner in the trace representation of A. If G is a (non-abelian) finite group, the structure of ideals in A^G was investigated in [**7**] by N. Riedel. Let τ be the unique tracial state on a UHF-algebra. Since the trace τ is α-invariant, the C^*-dynamical system (A, G, α) extends to the W^*-dynamical system $(\pi_\tau(A)'', G, \tilde{\alpha})$ where π_τ is the G. N. S. representation associated with τ. We set $K=\{g\in G;\ \tilde{\alpha}_g$ is an inner automorphism of $\pi_\tau(A)''\}$. Let $\hat{K}$ be the dual object of K. Since K is a normal subgroup of G, we obtain a G-space $(G, \hat{K})$ with the action,

$$(g\pi)(k)=\pi(g^{-1}kg)$$

for $k\in K$, $g\in G$ and $\pi\in\hat{K}$. By giving an equivalence $\sim$ by $\pi\sim\rho$ $(\pi, \rho\in\hat{K})$ iff $g\pi=\rho$ for some $g\in G$, we have an orbit space $\hat{K}/\sim$ (denoted by $\hat{K}/G$).

In this note we show that the number of extremal traces on the fixed point algebra A^G is the cardinality of the orbit space $\hat{K}/G$ and we give some conditions under which A^G is a UHF-algebra.

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2. Main results.

Let A_n be a matrix factor and π_n be a unitary representation of a finite group G into A_n. We define an action α of G on a UHF-algebra $A\equiv\otimes_{n=1}^{\infty}A_n$ by $\alpha_g=\otimes_{n=1}^{\infty}\mathrm{Ad}\pi_n(g)$.

We assume throughout that the automorphisms α_g are not inner in A except $g=e$, the unit in G.