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On polarized manifolds of Δ-genus two; part I

By Takao FUJITA

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Introduction.

By a polarized manifold we mean a pair (M, L) of a projective manifold M and an ample line bundle L on M. Set $n=\dim M$, $d(M, L)=L^n$ and $\Delta(M, L)=n+d(M, L)-h^0(M, L)$. Then $\Delta(M, L)\geqq 0$ for any polarized manifold (M, L) (see [**F2**]). We have classified polarized manifolds with $\Delta=0$ in [**F2**] and those with $\Delta=1$ in [**F5**] (as for positive characteristic cases, see [**F6**]). In this series of papers we will study polarized manifolds with $\Delta=2$. However, because of various technical reasons, we assume here that things are defined over the complex number field $\boldsymbol{C}$, although some arguments work in positive characteristic cases too.

This series is an improved version of [**F1**], which contains most results here, but, unfortunately, is hardly readable. We remark that Ionescu [**I**] obtained independently the classification of (M, L) with $\Delta=2$ such that L is very ample.

§ 0. Outline of the classification.

In this section we give a brief account of the classification of polarized manifolds with $\Delta=2$. We freely use the notation in [**F2**], [**F5**], [**F6**], etc. The following result is used to reduce various problems to lower dimensional cases.

(0.1) THEOREM. *Let (M, L) be a polarized manifold with* $\dim M=n\geqq 3$, $d(M, L)=d\geqq 2$ *and* $\Delta(M, L)=2$. *Then any general member D of $|L|$ is non-singular. Moreover, the restriction homomorphism $r: H^0(M, L)\to H^0(D, L_D)$ is surjective and $\Delta(D, L_D)=2$.*

PROOF. [**F7**; (4.1)] shows that D is smooth. If r is not surjective, we have $H^1(M, \mathcal{O}_M)>0$ and $\Delta(D, L_D)<2$. The latter implies $H^1(D, L_D)=0$ by [**F2**] and [**F5**]. This is absurd because we have an exact sequence $H^1(M, -L)\to H^1(M, \mathcal{O}_M)\to H^1(D, \mathcal{O}_D)$ and $H^1(M, -L)=0$ by Kodaira's vanishing theorem. Thus r is surjective and hence $\Delta(D, L_D)=2$.

(0.2) THEOREM. *Let (M, L) be a polarized manifold with* $\dim M=n\geqq 2$, $\Delta(M, L)=2$ *and $g(M, L)\leqq 1$, where $g(M, L)$ is the sectional genus. Then* $M\cong\boldsymbol{P}(E)$