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On spectral geometry of Kaehler submanifolds

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Introduction.

Let $x: M^n \to E^N$ be an isometric immersion of a compact Riemannian manifold into the Euclidean space. Let Δ be the Laplace-Beltrami operator of M acting on differentiable functions and $\mathrm{Spec}(M)=\{0<\lambda_1 \cdots \lambda_1<\lambda_2 \cdots \lambda_2< \cdots\}$ the spectrum of Δ, where each eigenvalue is repeated as many times as its multiplicity indicates. If $\phi: M \to E^N$ is a differentiable mapping we put $\phi=(\phi^1, \cdots, \phi^N)$, where ϕ^i is the i-th coordinate function of ϕ, and $\Delta\phi=(\Delta\phi^1, \cdots, \Delta\phi^N)$. We have the decomposition $x=\sum_k x_k$, $k \in \boldsymbol{N}$, where $x_k: M \to E^N$ is a differentiable mapping, $\Delta x_k=\lambda_k x_k$, and the addition is convergent, componentwise, for the L^2-topology on $C^\infty(M)$. Moreover x_0 is a constant mapping (it is the center of gravity of M) and $\{x_k\}_k$ are orthogonal mappings, that is

$$\int_M g(x_k, x_r)=0 \qquad \text{for all} \quad k, r, \quad k \neq r,$$

where g is the Euclidean metric on E^N. We have the relations

$$\Delta x=-nH=\sum_{k \geqq 1} \lambda_k x_k,$$

$$\Delta^2 x=-n\Delta H=\sum_{k \geqq 1} \lambda_k^2 x_k,$$

where H is the mean curvature vector of M in E^N. Let $k_1, k_2 \in \boldsymbol{N}$, $0 \leqq k_1<k_2$. We say that *the immersion x is of order* $\{k_1, k_2\}$ if

$$x_k=0 \qquad \text{for all} \quad k \in \boldsymbol{N}, \quad k \neq 0,\ k_1 \text{ or } k_2.$$

If $k_1=0$ we say simply that *the immersion is of order* k_2.

It is well-known that the complex projective space, $\boldsymbol{C}P^m$, with the Fubini-Study metric, admits an isometric imbedding of order 1 in the Euclidean space, which has parallel second fundamental form. In [**11**] S. Tai gives a simple version of this one. From this fact we can view any submanifold, M^n, of $\boldsymbol{C}P^m$ as a submanifold of the Euclidean space, $x: M^n \to E^N$. In [**10**] we have obtained some information about the spectral geometry of submanifolds in the complex projective space, studying Δx and the order k immersion. In this paper