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# Complex crystallographic groups II

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## §0. Introduction.

Let $\boldsymbol{E}(n)$ be the complex motion group acting on the $n$-dimensional complex euclidean space $X \cong \boldsymbol{C}^n$. A complex crystallographic group is, by definition, a discrete subgroup of $\boldsymbol{E}(n)$ with compact quotient. In a previous paper [**6**], we studied general properties of the quotient varieties and determined all the two dimensional crystallographic reflection groups.

In this paper, we treat two dimensional complex crystallographic group $\Gamma$ such that the quotient variety $M=X/\Gamma$ is biholomorphic to the two dimensional projective space $\boldsymbol{P}^2$. We list up all such groups (Theorem 1). Generators and fundamental relations are obtained (Theorem 2). Let $\phi$ denote the natural mapping: $X \rightarrow M$. The coordinate representation of $\phi$, the branching locus $D$ and the ramification indices of $\phi$ on $D$ are determined (Theorem 3). We explicitly give the representation $h: \pi_1(M-D) \rightarrow \Gamma$ and the kernel of $h$ (Theorem 4).

## §1. Notations and definitions.

The unitary group of size 2 is denoted by $U(2)$. For $A \in U(2)$ and $a \in \boldsymbol{C}^2$, $(A \mid a) \in \boldsymbol{E}(2)$ denotes the transformation: $x \rightarrow Ax+a$. For a two dimensional complex crystallographic group $\Gamma$,

$$L := \{a\,;\,(1 \mid a) \in \Gamma\}$$

and

$$G := \{A\,;\,(A \mid a) \in \Gamma\}$$

are called the lattice and the point group of $\Gamma$, respectively. If $\Gamma$ has the representation $\{(A \mid a);\ A \in G,\ a \in L\}$, then we call $\Gamma$ the semidirect product $G \ltimes L$ of the lattice and the point group.

DEFINITION. Imprimitive reflection group $G(m, p, 2) \subset U(2)$ is the group generated by

$$\begin{pmatrix} & 1 \\ 1 & \end{pmatrix},\ \begin{pmatrix} & \theta \\ \theta^{-1} & \end{pmatrix} \text{ and } \begin{pmatrix} \theta^p & \\ & 1 \end{pmatrix},\quad \theta = \exp\frac{2\pi\sqrt{-1}}{m}.$$

DEFINITION. An element $g \in \boldsymbol{E}(2)$ is called a reflection if $g$ is of finite order, $g \neq$ identity and keeps a line $H(g) \subset X$ pointwise fixed.

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