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A finite difference approach to the number of peaks of solutions for semilinear parabolic problems

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Introduction.

In this paper we study the number of peaks of solutions for one-dimensional semilinear parabolic problems by a finite difference method. As a model problem let us consider the equation in $u=u(x, t)$.

$$(0.1)\qquad \begin{cases} u_t=u_{xx}+f(u), & 0<x<1 \quad \text{and} \quad t>0, \\ u_x(0, t)=u_x(1, t)=0, & t>0, \\ u(x, 0)=u^0(x), & 0<x<1, \end{cases}$$

where f is a smooth function. We have two purposes; one is to know how the number of peaks of $u(\cdot, t)$ changes as t passes, the other is to present a finite difference scheme for (0.1) whose solution has the same behavior as the exact one concerning the number of peaks. Our result for the former is that the number of peaks is monotonically decreasing. We do not prove it independently of the latter. But we first attack the latter and present a finite difference scheme whose solution has the monotonically decreasing property with regard to the number of peaks. After that we prove the above result by the limit process.

Thus our main effort is devoted to constructing a finite difference scheme whose solution has the property mentioned above under appropriate conditions. Of course, it should also be shown that the finite difference solutions converge to the exact one as h and τ (space mesh and time mesh) tend to zero. In our scheme for (0.1), roughly speaking, the condition $\tau/h^2\leqq 1/2$ yields the convergence, while the condition $\tau/h^2<1/4$ leads to the property in question (Remark 2.7).

As a simple application of our result let us show a consequence relating to the stability of equilibrium solution. Chafee [1] and Matano [5] showed that every nonconstant equilibrium solution of (0.1) is unstable, while Ito [3] proved that for each (unstable) equilibrium solution there exists a stable manifold such that the solution of (0.1) starting from any function on the