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On the hyperplane section principle of Lefschetz

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The purpose of this article is to describe various versions of the following principle:

Let A be an ample divisor on a manifold M. Then the structure of M is closely related to that of A.

This was the philosophy of the study of Lefschetz on the hyperplane sections. Recently Sommese [24] developed various techniques in the above spirit and he found many examples of manifolds A which cannot be ample divisors in any manifold M because the above 'relation' implies so severe conditions on M that they cannot be compatible with each other. Here, inspired also by the work [17] of Mori, we develop further the methods of Sommese, improve some of his results, answer to questions and conjectures raised by him, and give several new examples. We shall find the tools thus obtained to be very powerful in the study of polarized varieties (see [5], [6]).

In this paper we work in the category of algebraic spaces defined over an algebraically closed field K of any characteristic (However, in the statements indicated by /$\boldsymbol{C}$, K is assumed to be the complex number field $\boldsymbol{C}$. For example, $K=\boldsymbol{C}$ in § 1, but (2.1) is valid in positive characteristic cases too.). In § 1 we review the classical Lefschetz theory. § 2 is devoted to the study of various types of extension theorems from A to M, for cohomologies, line bundles, linear systems, morphisms and so on. In § 3 we consider the case in which A is isomorphic to a complete intersection in a projective space. In § 4 we study the case in which A is a fiber bundle over a manifold. § 5 is for the case where A is a blowing up of another manifold. Two appendixes about a couple of techniques in this paper are added for the convenience of the reader.

Our results in the case char $K>0$ are far from satisfactory because of the lack of a vanishing theorem of Kodaira type. However, the author expects that our principle itself is of great importance in positive characteristic cases too.

NOTATION, CONVENTION and TERMINOLOGY.

Variety means an irreducible reduced algebraic space which is assumed to be proper over K unless otherwise stated explicitly. Smooth means non-