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Notes on infinitesimal variations of submanifolds

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§ 0. Introduction.

In a previous paper [5], the present author studied variations of the metric tensor, the Christoffel symbols and the second fundamental tensors of submanifolds of a Riemannian manifold under infinitesimal variations of the submanifolds.

In this paper, we assume that submanifolds under consideration are compact and orientable and we obtain, using integral formulas, some global results on infinitesimal isometric, affine and conformal variations of the submanifolds.

§ 1. Preliminaries [1].

We consider an m-dimensional Riemannian manifold M^m covered by a system of coordinate neighborhoods $\{U; x^h\}$ and denote by g_{ji}, Γ^h_{ji} and ∇_j the metric tensor, the Christoffel symbols formed with g_{ji} and the operator of covariant differentiation with respect to Γ^h_{ji} of M^m respectively, where, here and in the sequel, the indices $h, i, j, k, \cdots$ run over the range $\{1', 2', \cdots, m'\}$.

We then consider an n-dimensional compact orientable Riemannian manifold M^n covered by a system of coordinate neighborhoods $\{V; y^a\}$ and denote by g_{cb}, Γ^a_{cb}, ∇_c, $K_{dcb}{}^a$ and K_{cb} the metric tensor, the Christoffel symbols formed with g_{cb}, the operator of covariant differentiation with respect to Γ^a_{cb}, the curvature tensor and the Ricci tensor of M^n respectively, where, here and in the sequel, the indices $a, b, c, \cdots$ run over the range $\{1, 2, \cdots, n\}$.

We assume that M^n is isometrically immersed in M^m by the immersion: $M^n \to M^m$ and represent the immersion by

$$x^h = x^h(y^a).$$

Since the immersion is isometric, we have

$$g_{cb} = B_c{}^j B_b{}^i g_{ji}, \tag{1.1}$$

where we have put $B_c{}^j = \partial_c x^j$ $(\partial_c = \partial/\partial y^c)$.

We can assume that $[B_b{}^h]$ gives the positive orientation of M^n.