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# Construction of a solution of random transport equation with boundary condition

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## § 0. Introduction.

Concerning the analysis of wave propagation in random media, S. Ogawa [7] introduced a new type of partial differential equation of first order with a random coefficient:

$$(0.1)\qquad \frac{\partial u}{\partial t}(t, x; \omega)+\{\dot{B}_t(\omega)+b(t, x)\}\frac{\partial u}{\partial x}(t, x; \omega)$$
$$=c(t, x)u(t, x; \omega)+d(t, x),$$
$$(t, x)\in[0, T]\times R^1, \qquad T<\infty,$$

where $\dot{B}_t(\omega)$ is the white noise. He constructed a solution of Cauchy problem of equation (0.1) with given initial data

$$(0.2)\qquad u(0, x; \omega)=\phi(x).$$

His main tools are a stochastic integral which he defined and the concept of the differentiation $\dfrac{\partial X_t}{\partial B_t}$ of a stochastic process $X_t$ with respect to the Brownian motion $B_t$.

Here, in this paper, we consider a natural extension of his equation:

$$(0.3)\qquad \frac{\partial u}{\partial t}(t, x; \omega)+\sum_{i,j=1}^{d}\{a_{ij}(t, x)\dot{B}_t^j(\omega)+b_i(t, x)\}\frac{\partial u}{\partial x_i}(t, x; \omega)$$
$$=c(t, x)u(t, x; \omega)+d(t, x),$$
$$(t, x)\in[0, T]\times G,$$

with initial data (0.2) and boundary conditions at $\partial G$, where $G$ is a given region in $R^d$ $(d\geqq 1)$ and $\dot{B}_t(\omega)=\{\dot{B}_t^j(\omega)\}_{j=1}^d$ is the $d$-dimensional white noise. More precisely, we construct a solution of the equation (0.3) for