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Integration of analytic differential systems with singularities and some applications to real submanifolds of C^n *

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1. Introduction.

A module D of analytic vector fields on $\boldsymbol{R}^n$ defines at each $y \in \boldsymbol{R}^n$ a subspace $D(y)=\{X(y): X \in D\}$ of the tangent space to $\boldsymbol{R}^n$ at y. A real-analytic submanifold M of $\boldsymbol{R}^n$ is an *integral manifold of* D if

$$T_y M = D(y) \qquad \text{for all} \quad y \in M, \tag{1.1}$$

where $T_y M$ is the tangent space to M at y. In [6] Nagano proved that if D is closed under the Lie bracket then through each point there passes a unique integral manifold of D. This result extends the classical Frobenius theorem, which assumes in addition that $D(y)$ has constant dimension. In dropping this hypothesis, Nagano relies on the analyticity. The classical theorem also holds in the C^∞ category and in [6] Nagano gives a simple C^∞ counterexample to his result.

This paper contains 1) a new proof of Nagano's theorem in a formulation which describes the integral manifold directly in terms of D, 2) a sharpened form of the theorem giving necessary and sufficient conditions at p for the existence of an integral manifold through p, and 3) some applications of these results to the local geometry of real-analytic submanifolds of a complex manifold. In particular, it is shown that a point p on a real-analytic CR submanifold M is not of finite weight [1] if and only if there is a complex submanifold of M of maximum dimension through p.

The proofs given here are almost entirely algebraic and make no use of differential equations. They appear to be new even in the classical case where $D(y)$ has constant dimension. Besides a simple and standard majorization argument and advanced calculus, one needs only the standard Weierstrass division theorem [3, Satz 1, p. 23]. All definitions are within the real-analytic category,

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