A characterization of the Rudvalis group

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The purpose of this paper is to prove the following:

THEOREM A. *Simple groups with Sylow 2-subgroups of type Rd are isomorphic to Rd, where Rd is the Rudvalis simple group of order $2^{14} \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 13 \cdot 29$***.

This is a Corollary of the following result:

THEOREM B. *Let G be a finite group with Sylow 2-subgroup T satisfying the following condition:*

(a) *T is of order at least 2^{14};*

(b) *$Z_4(T)$ is of order 16;*

(c) *$\Phi(W)$ is of order at most 8, where $W=C_T(Z_3(T))$;*

(d) *all subgroups of W of index at most 16 contain $\Phi(W)$.*

Assume further that G has no subgroup of index 2. Then one of the following holds:

(1) *$G/O(G)$ is isomorphic to the Rudvalis group.*

(2) *$O(G)W$ is normal in G and $G/O(G)W$ is isomorphic to $GL(3, 2)$.*

Our notation is standard and taken from [5].

1. Transfer.

Except for Lemma 1.4, we assume that G is a finite group with Sylow 2-subgroup T satisfying the conditions (a) to (d) in Theorem B and we set $W=C_T(Z_3(T))$.

LEMMA 1.1. *The following hold:*

(1) $Z(W) \geqq W' = \Phi(W) = Z_3(T) \cong Z_2^3$;

(2) $|T| = 2^{14}$, $T/W \cong D_8$, $W/W' \cong Z_2^8$, $T/W' \cong Z_2 \wr D_8$;

(3) *$N_G(T)$ has a normal 2-complement;*

(4) *If $|W : W \cap T^g| \leqq 2$ for $g \in G$, then $g \in N_G(W')$;*

(5) *W is weakly closed in T with respect to G.*

**) This theorem is proved by Assa, too.