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Algebraic differential equations of the first order free from parametric singularities from the differential-algebraic standpoint

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Abstract.

A *differential-algebraic* definition for an algebraic differential equation of the first order to be free from parametric singularities will be given. From this standpoint we shall prove three theorems which are essentially due to Briot and Bouquet, Fuchs, and Poincaré respectively.

§ 0. Introduction.

Let k be an algebraically closed differential field of characteristic 0, and K be a one-dimensional algebraic function field over k. We shall suppose that K is a differential extension of k. Let P be a prime divisor of K, and K_P be the completion of K with respect to P. Then, K_P is a differential extension of K, and the differentiation gives a continuous mapping from K_P to itself (cf. [8]).

We shall say that K is a *differential algebraic function field* over k if there exists an element y of K such that $K=k(y, y')$. Suppose that K is a differential algebraic function field over k. Then, y and y' satisfy an irreducible algebraic equation $F(y, y')=0$ over k. We shall say that K is associated with F in y.

Conversely, let $k\{y\}$ be the differential polynomial algebra in a single indeterminate y over k. We shall take and fix a universal extension Ω of k, the existence of which was proved by Kolchin [3, p. 771]. Let F be an algebraically irreducible element of $k\{y\}$ of the first order, and η be a generic point of the general solution of F in Ω over k. Then, $k(\eta, \eta')$ is a differential algebraic function field over k associated with F in η.

Throughout this note K will denote a differential algebraic function field over k.

Let ν_P be the normalized valuation in K belonging to a prime divisor P of K. If $\nu_P(\tau')\geqq 0$ for a prime element τ in P, then $\nu_P(\sigma')\geqq 0$ for any prime ele-

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