J. Math. Soc. Japan
Vol. 30, No. 3, 1978

Bounded, periodic and almost periodic classical solutions of some nonlinear wave equations with a dissipative term

By Mitsuhiro NAKAO

(Received Feb. 3, 1976)
(Revised June 20, 1977)

Introduction.

This paper deals with the questions of the existence and asymptotics of the bounded, periodic and almost periodic classical solutions for the equations

$$\text{(E)} \qquad \frac{\partial^2}{\partial t^2}u+\sum_{|\alpha|,|\beta|\leqq m}(-1)^{|\alpha|}D^{\alpha}(a_{\alpha,\beta}(x)D^{\beta}u(x,t))+\nu\frac{\partial}{\partial t}u$$
$$+F(x,t,u)=0 \qquad \text{on} \quad \Omega\times R \text{ (or } \Omega+R^{+})$$

together with the boundary condition

$$\text{(B)} \qquad D^{\alpha}u|_{\partial\Omega}=0 \qquad \text{for} \quad |\alpha|\leqq m-1,$$

where ν is a positive constant, Ω is a bounded domain in n-dimensional Euclidean space R^n and $\partial\Omega$ its boundary. We use the following notations

$$\alpha=(\alpha_1,\cdots,\alpha_n), \quad |\alpha|=\sum_{i=1}^{n}|\alpha_i|, \quad D^{\alpha}=\prod_{i=1}^{n}\left(\frac{\partial}{\partial x_i}\right)^{\alpha_i},$$
$$x=(x_1,\cdots,x_n) \quad \text{etc.}$$

The functions to be considered are all real valued and throughout the paper we make the following assumptions:

H_1. $$A=\sum_{|\alpha|,|\beta|\leqq m}(-1)^{|\alpha|}D^{\alpha}(a_{\alpha,\beta}(x)D^{\beta}u)$$

is formally selfadjoint and coercive on $\mathring{H}_m$, i.e., there exist some positive constants c_0, c_1 such that

$$c_1^2\|u\|^2_{\mathring{H}_m}\geqq\langle Au,u\rangle\geqq c_0^2\|u\|^2_{\mathring{H}_m} \qquad \text{for} \quad u\in\mathring{H}_m$$

where we put