J. Math. Soc. Japan
Vol. 30, No. 1, 1978

Induced characters of some 2-groups

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(Received May 6, 1976)

Let G be a 2-group and χ a complex, irreducible charactor of G. The Schur index of χ with regard to the rational field $\boldsymbol{Q}$ is denoted by $m_{\boldsymbol{Q}}(\chi)$. It is known that $m_{\boldsymbol{Q}}(\chi)=1$ or 2, and that if $m_{\boldsymbol{Q}}(\chi)=2$, then there exist a subgroup H of G and an irreducible character ϕ of H such that χ is induced from ϕ, i.e., $\chi=\phi^{G}$, $m_{\boldsymbol{Q}}(\phi)=2$, and the factor group H/N, $N=$kernel of ϕ, is a generalized quaternion group (cf. (11.7) and (14.3) of [2], or [3]).

Now, let H' be a generalized quaternion group. The faithful irreducible characters of H' are algebraically conjugate to each other and their Schur indices are equal to 2, whereas any non-faithful irreducible character of H' has Schur index 1 (cf. [5, § 6]). So we ask a question: Let G be a 2-group, let H, N be subgroups of G such that $H \triangleright N$ and H/N is a generalized quaternion group, and let ϕ be a faithful irreducible character of H/N, which is also regarded as a character of H. Suppose that the induced character ϕ^{G} is irreducible. Is it true that the Schur index $m_{\boldsymbol{Q}}(\phi^{G})=2$?

A simple case for the question is that $N=\{1\}$. Namely, let $G\supset H$ be 2-groups, where H is a generalized quaternion group, and let ϕ be an irreducible charater of H such that $m_{\boldsymbol{Q}}(\phi)=2$ and ϕ^{G} is irreducible. Is it ture that $m_{\boldsymbol{Q}}(\phi^{G})=2$? The purpose of the paper is to show that this is true for a class of induced characters ϕ^{G}, which are associated with *cyclotomic algebras*. Our result yields, as a special case, that if $[G: H]=2$, then the question is affirmative.

Let us briefly explain the contents of the paper. From now on, H_{n} denotes the generalized quaternion group of order 2^{n+1} $(n\geqq 2)$, ϕ_{n} an irreducible character of H_{n} with $m_{\boldsymbol{Q}}(\phi_{n})=2$, and ζ_{s} a primitive s-th root of unity, where s is a natural number. In § 1, we investigate a 2-group G such that $G\supset H_{n}$ and that the induced character ϕ_{n}^{G} is irreducible (Theorem 1). We also determine the values of ϕ_{n}^{G} at elements x of G (Proposition 1).

In § 2, we study a cyclotomic algebra B made with the extension $\boldsymbol{Q}(\zeta_{2^n})/k$, where k is a subfield of the field $\boldsymbol{Q}(\zeta_{2^n})$ of 2^{n}-th roots of unity. It will be shown that the index of B is 1 or 2, and if B has index 2, then there exists a 2-group G, which is a finite subgroup of the multiplicative group $B^{\times}$ such that