J. Math. Soc. Japan
Vol. 26, No. 4, 1974

Compact two-transnormal hypersurfaces in a space of constant curvature*)

By Seiki NISHIKAWA

(Received March 12, 1973)
(Revised Nov. 19, 1973)

Introduction.

Let M be a complete Riemannian n-manifold isometrically imbedded into a complete Riemannian $(n+1)$-manifold W. Throughout this paper manifolds are always assumed to be connected and smooth. Furthermore we assume $n \geqq 2$, although some of our results are valid even for $n=1$. For each $x \in M$ there exists, up to parametrization, a unique geodesic τ_x of W which cuts M orthogonally at x. M is called a *transnormal hypersurface* of W if, for each pair $x, y \in M$, the relation $\tau_x \ni y$ implies that $\tau_x = \tau_y$, i.e. if each geodesic of W which cuts M orthogonally at some point cuts M orthogonally at all points of intersection. As is well-known, every surface of constant width in the ordinary Euclidean space has this property ([**6**]), and it is a model of a transnormal hypersurface.

The order of a transnormal hypersurface, by which the hypersurface is globally characterized, is introduced in the following way. Define an equivalence relation $\sim$ on M by writing $x \sim y$ to mean $y \in \tau_x$. With respect to this relation, take the quotient space $\hat{M} = M/\sim$ and endow $\hat{M}$ with the quotient topology. We call M an *r-transnormal* hypersurface if the natural projection ψ of M onto $\hat{M}$ is an r-fold (topological) covering map. The number r is called the *order of transnormality* of M. It should be remarked that ψ is not always a covering map. However, if W is simply connected and of constant curvature, then ψ is a covering map ([**5**]).

In [**5**], we have obtained the following results which determine topological structures of transnormal hypersurfaces.

THEOREM A. *Let M be an n-dimensional transnormal hypersurface of W. Suppose that there exists a point p of M whose cut locus $C(p)$ in W does not intersect M: $C(p) \cap M = \emptyset$. Then the following hold.*

(i) *If M is 1-transnormal, then M is homeomorphic to a Euclidean n-space E^n.*

*) This paper was written while the author was at Tokyo Metropolitan University.