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A note on complex K-theory of infinite CW-complexes

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In the present paper we study certain properties of the complex cohomology and homology K-theories K^* and K_* defined on the category of based CW-complexes (or of the same homotopy type).

There exists a universal coefficient sequence

$$0 \longrightarrow \mathrm{Ext}(\tilde{K}_n(X), Z) \longrightarrow \tilde{K}^{n+1}(X) \longrightarrow \mathrm{Hom}(\tilde{K}_{n+1}(X), Z) \longrightarrow 0$$

between $\tilde{K}^*$ and $\tilde{K}_*$ [**9**]. So we can define a duality homomorphism $D: \chi(\tilde{K}_n(X)) \to \tilde{K}^{n+1}(X)$ by the composition

$$\chi(\tilde{K}_n(X)) \longrightarrow \mathrm{Ext}(\tilde{K}_n(X), Z) \longrightarrow \tilde{K}^{n+1}(X)$$

where $\chi(\tilde{K}_n(X))$ is the character group of the discrete abelian group $\tilde{K}_n(X)$. We give a necessary and sufficient condition that the duality homomorphism D is an isomorphism (Theorem 1). This theorem contains Vick's result [**7**] as a corollary.

Anderson-Hodgkin [**1**] computed the K^*-groups of the Eilenberg-MacLane spaces $K(\pi, n)$ for certain countable abelian groups π. The purpose of the present paper is to remove the countability restriction. First we dualize Anderson-Hodgkin's Theorem for a countable abelian group using the above universal coefficient sequence, and extend the dualized result to an arbitrary abelian group (Theorem 2). Then, dualizing it again we show the result of Anderson-Hodgkin without the assumption on the cardinality of an abelian group (Theorem 3).

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§ 1. A duality homomorphism.

1.1. Let $\tilde{K}^*$ and $\tilde{K}_*$ be the Z_2-graded reduced complex cohomology and homology K-theories represented by the unitary spectrum, which are defined on the category of based CW-complexes (or of the same homotopy type). We notice that $\tilde{K}^*$ and $\tilde{K}_*$ are additive and of finite type. $\tilde{K}^*$ and $\tilde{K}_*$ are related by the following universal coefficient sequence [**9**]: There exists a