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On the ε-entropy of stable processes

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§ 1. Introduction and main result.

In recent years many researches are presented about "*rate distortion theory*" or "*ε-entropy*" or "*information rate*" in certain branches of information theory. They are largely motivated by practical problems—for example, data compression or coding of signals. The notion of ε-entropy is originally due to C. Shannon and is closely connected with his fundamental theorem. We can regard the ε-entropy of a stochastic process as a characteristic quantity of the process—a certain index of complexity of the process in view of finite-dimensional approximation.

It will be important to carry out estimates of ε-entropy for basic stochastic processes. For Gaussian processes Pinsker showed how to estimate the ε-entropy [**5**]. Many other researches are concentrated on the discussions about Gaussian cases. On the other hand, we know few estimates for non-Caussian processes: for a diffusion process [**1**] and for a process of jumping type with discrete state space [**3**].

In the present paper we give an estimate for stable processes.

Let $(\Omega, \mathcal{B}, P)$ be a probability space. For random variables (stochastic processes) ξ, η, ζ etc., whose state spaces might be different measurable spaces, Kolmogorov defined the amount of information $I(\xi, \eta)$ and the average conditional information $EI(\xi, \eta|\zeta)$. Though in the definition of $EI(\xi, \eta|\zeta)$ E is simply a symbol and has no meaning of expectation, in our cases we may consider it equal to the expectation of information between ξ and η under the condition with respect to ζ. We list up several properties of them which we shall use in the sequel without mention. We omit assumptions necessary for the formulas since we shall deal with only the cases when the assumptions are satisfied. We refer readers to [**4**] for assumptions and terminologies which we do not define here.

a) $I(\xi, \eta)=0$ if and only if ξ and η are independent.

b) If (ξ_1, η_1) and (ξ_2, η_2) are independent, then

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