J. Math. Soc. Japan
Vol. 26, No. 1, 1974

Pinching theorem for the real projective space

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(Received Dec. 20, 1972)
(Revised May 17, 1973)

§ 1. Introduction.

Let M be an n-dimensional connected and complete Riemannian manifold whose sectional curvature K satisfies

(1.1) $$1/4 < \delta \leqq K \leqq 1 \qquad \text{for any plane section}.$$

If M is simply connected and $\delta \doteqdot 0.85$, then M is diffeomorphic to the standard sphere (see [5]). In the present paper we shall establish a differentiable pinching theorem for the real projective space. Our pinching number is independent of the dimension.

MAIN THEOREM. *Let M be a connected and complete Riemannian manifold with* (1.1). *Assume that the fundamental group $\pi_1(M)$ of M is*

(1.2) $$\pi_1(M) = Z_2 .$$

Then there exists a constant $\delta_0 \in (1/4, 1)$ such that

(1.3) $$\delta > \delta_0$$

implies M to be diffeomorphic to the real projective space.

§ 2. Preliminaries.

Throughout this paper, let M satisfy both (1.1) and (1.2). We denote by d the distance function on M with respect to the Riemannian metric. The diameter $d(M)$ of M is defined by $d(M) := \mathrm{Max}\,\{d(x, y)\,;\, x, y \in M\}$ and we set $d(p, q) := d(M)$. Let $\tilde{M}$ be the universal Riemannian covering manifold of M and π the covering projection. For any point $x \in M$, we denote by $\tilde{x}_1$, $\tilde{x}_2 \in \tilde{M}$ the elements of the inverse image $\pi^{-1}(x)$, and by $C(x)$ the cut locus of x. Under the assumptions (1.1) and (1.2), we see in [4] that

$$\pi/2 \leqq d(x, C(x)) \leqq \pi/2\sqrt{\delta}\,, \qquad \pi/2 \leqq d(M) \leqq \pi/2\sqrt{\delta}$$

hold for any $x \in M$. Since for any $x \in M$ and any $y \in C(x)$, each minimizing geodesic from x to y has no conjugate pair, they are joined by two and just two distinct minimizing geodesics. Let E be defined by