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On certain nonlinear evolution equations

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§ 1. Introduction.

We consider nonlinear evolution equations of the form

$$\frac{d}{dt}u(t)+A(t)u(t)\ni f(t, u(t)), \qquad 0\leqq t\leqq T$$

in a real Hilbert space H. Here, for each fixed t, $A(t)$ is a (possibly) multi-valued nonlinear operator in H of the form $\partial\varphi^t$ (subdifferential of a lower semicontinuous convex function φ^t from H into $(-\infty, \infty]$, $\varphi^t\not\equiv\infty$), while $f(t, \cdot)$ is continuous from the strong to the weak topology of H and $f(t, \cdot)-\beta(t)\cdot$ is dissipative in H for some Lebesgue integrable function β on $[0, T]$.

We denote the inner product and the norm in H by $(\ , \ )$ and $\| \ \|$ respectively. Let φ be a lower semicontinuous convex function from H into $(-\infty, \infty]$. The effective domain of φ is defined by $\{u\in H;\ \varphi(u)<\infty\}$. The subdifferential $\partial\varphi$ of φ is defined by $\partial\varphi(u)=\{w\in H;\ \varphi(v)\geqq\varphi(u)+(w, v-u)$ for all $v\in H\}$ for each $u\in H$, and the domain of the subdifferential $\partial\varphi$ is defined by $D(\partial\varphi)=\{u\in H;\ \partial\varphi(u)\neq\emptyset\}$.

Let T be a positive constant. For each $t\in[0, T]$, let φ^t be a lower semicontinuous convex function from H into $(-\infty, \infty]$ with nonvoid effective domain, and suppose that $\{\varphi^t;\ 0\leqq t\leqq T\}$ satisfies the following three conditions:

(I) *The effective domain D of φ^t is independent of t.*

(II) *For every $r>0$, there exist two positive constants c_r and c_r' such that*

$$|\varphi^s(u)-\varphi^t(u)|\leqq|s-t|\cdot[c_r\varphi^t(u)+c_r']$$

holds if $0\leqq s, t\leqq T$, $u\in D$ and $\|u\|\leqq r$.

(III) *For some $b\in D$, b is in $D(\partial\varphi^t)$ for almost all $t\in[0, T]$ and $|||\partial\varphi^t(b)|||\equiv\min\{\|v\|;\ v\in\partial\varphi^t(b)\}$ is Lebesgue integrable in $0\leqq t\leqq T$.* (See Corollary 3.4.)

Let f be a map of $[0, T]\times H$ into H, and suppose that f satisfies the following three conditions:

(IV) *For each fixed $u\in H$, $f(t, u)$ is strongly measurable in $0\leqq t\leqq T$, and for each fixed $t\in[0, T]$ it is continuous from the strong to the weak topology of H.*