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On conformal diffeomorphisms of complete Riemannian manifolds with parallel Ricci tensor

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We shall assume, throughout this paper, that Riemannian manifolds are connected and of dimension >2, their metrics are positive-definite, and manifolds and diffeomorphisms are of differentiability class C^{∞}.

Let M and M^* be Riemannian manifolds with metric tensor g and g^* respectively, and f a diffeomorphism of M onto M^*. If the induced metric f^*g^* of g^* by f is related to g by

$$f^*g^*=\rho^{-2}g,$$

then f is called a *conformal diffeomorphism* of M onto M^*, where ρ is a positively valued scalar field on M. If the scalar field ρ satisfies the equation

$$\nabla\nabla\rho=\phi g,$$

∇ indicating covariant differentiation and ϕ being a scalar field, then the diffeomorphism f is called a *concircular* one, which carries Riemannian circles of M to those of M^*. If ρ is a constant or equal to 1, then f is said to be *homothetic* or *isometric*, respectively.

THEOREM 1. *Let M and M^* be complete Riemannian manifolds with parallel Ricci tensor. If there is a non-isometric conformal diffeomorphism f of M onto M^*, then f is homothetic or both the manifolds M and M^* are isometric to the sphere.*

THEOREM 2. *Let M be a complete Riemannian manifold with parallel Ricci tensor. If M admits a non-isometric conformal transformation f, then occurs one of the following two cases:*

(1) *f is homothetic and M is a Euclidean space, or*

(2) *M is isometric to the sphere.*

The first and principal part of Theorem 1 was proved by N. Tanaka [6] by a group-theoretic method under some additional conditions on Ricci tensors of M and M^*. Then T. Nagano [5] dealt with the cases excepted by Tanaka, showed that f is properly conformal only in the case where both M and M^* are Einstein manifolds of positive curvature, and completed Theorem