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On the imbedding problem of Galois extensions

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Introduction

Let Ω be a field, and k a finite Galois extension of Ω with Galois group $\mathfrak{g}=G(k/\Omega)$. Let $\varphi: G \to \mathfrak{g}$ be a homomorphism of a finite group G onto $\mathfrak{g}$ with kernel A. Then we have an exact sequence

$$1 \longrightarrow A \longrightarrow G \xrightarrow{\varphi} \mathfrak{g} \longrightarrow 1 . \qquad (1)$$

We say that the imbedding problem $(k/\Omega, G, \varphi)$ associated with the exact sequence (1) is solvable, if there exists a Galois algebra K*) over Ω with Galois group $\mathfrak{G}=G(K/\Omega)$ such that:

1) There is an isomorphism π of G onto $\mathfrak{G}$.
2) k is contained in K, and it is the fixed subalgebra of K under A^{π}.
3) φ is the composite of π with the naturally induced epimorphism of G onto $\mathfrak{g}$.

Such a K is said to be a solution of the imbedding problem. (For simplicity we shall write g instead of g^{π} for $g \in G$.)

We shall be concerned with the imbedding problem only when the following conditions are satisfied:

1) The group A is abelian.
2) The characteristic of the field Ω is relatively prime to the order of A.

The purpose of the present paper is to summarize some properties about the imbedding problem, as a preparation to prove the main theorem in the author's following paper.

§1. A necessary condition for the solvability of the imbedding problem

1.1. For $s \in \mathfrak{g}$ choose an element $g_s \in G$ such that

*) A commutative algebra K over Ω is called a Galois algebra with Galois group $\mathfrak{G}$, if the following conditions are satisfied: 1) K is semi-simple, 2) $\mathfrak{G}$ is a group of automorphisms of K over Ω, 3) K is isomorphic to the group ring $\Omega[\mathfrak{G}]$ as right $\mathfrak{G}$-modules. For the general theory of Galois algebras, see [**2**] and [**3**].