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On a special kind of Dirichlet series

Dedicated to Professor Shôkichi Iyanaga on his 60th birthday

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The purpose of the present paper is to introduce a very simple idea which leads to a special kind of zeta-function satisfying a functional equation of the usual form. At least according to the knowledge of the author, the zeta-function has never been investigated at any place, although the function lies entirely in the frame of the classical theory of number fields. For this purpose, we use two tools. One is the Eisenstein series in the sense of [**6**], [**7**], containing a character of a discontinuous group, and the other is a special kind of character, as studied in [**3**], of a discontinuous group of Hilbert's type. If a character of this kind, which has an intimate connection with non-congruence normal subgroups of arithmetically defined discontinuous groups, is used in the construction of an Eisenstein series, then we obtain Dirichlet series of a rather unfamiliar fashion from non-constant terms in the Fourier expansion of the Eisenstein series, and the functional equation of the zeta-function determined by the Dirichlet series follows immediately from the functional equation of the Eisenstein series. The coefficients of the Dirichlet series, i.e. numerators of individual terms in the usual notation, are in fact Gauss sums containing congruence characters that are not quadratic.

The space on which the Eisenstein series of our present interest are considered is, in general, a finite direct product of real, three dimensional hyperbolic space. On such a space we have a nice discontinuous operation of a modular group of Hilbert's type made up from a totally imaginary number field F. Since we need non-quadratic power residue symbols of the ground field F in the definition of the character χ, the field F must contain at least four roots of unity. Hence, it is enough to observe only totally imaginary fields.

Since, however, we intend in this paper a rapid explanation of the idea, we shall treat here only the simplest case of $F=\boldsymbol{Q}(\sqrt{-1})$. As discontinuous groups, we take subgroups of $SL(2, \mathfrak{o})$ where $\mathfrak{o}$ is the ring of integers of F. The space, on which such groups operate, is the three dimensional hyperbolic space $H=SL(2, \boldsymbol{C})/SU(2)$, so that all results, for example in [**5**], about discontinuous groups of Picard's type are applicable.