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On normal almost contact structures

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Introduction

In the present paper we shall investigate some relations between almost contact structures and complex structures. An odd dimensional differentiable manifold is said to be an almost contact manifold if there exists a triple $\Sigma=(\phi, \xi, \eta)$, ϕ being (1.1)-tensor field, ξ a vector field and η 1-form on M satisfying certain conditions. We shall introduce in § 2 an almost complex structure on the product manifold of two almost contact manifolds. We shall call an almost contact structure on M to be integrable if the induced almost complex structure on $M\times M$ is integrable (i. e., complex structure). In § 2 we shall prove that the induced almost complex structure on the product of two almost contact manifolds is integrable if and only if both almost contact structures are integrable. This result generalises a theorem of Calabi-Eckmann which says that the product of two odd dimensional spheres admits a complex structure [3]. This result says also that the notion of the integrable almost contact structure coincides with that of the normal almost contact structure defined by Sasaki-Hatakeyama [8].

In § 3 we shall introduce the notion of isomorphism and automorphism of almost contact structures and we shall prove that the automorphism group of a compact almost contact manifold M is a Lie transformation group of M with respect to the compact-open topology, if the structure is integrable.

In § 4 we shall show examples of compact normal almost contact manifolds other than the odd dimensional spheres. In particular, we shall see that every compact simply connected homogeneous contact manifold studied by Boothby-Wang [2] has always a normal almost contact structure such that the automorphism group operates transitively.

In the last section we shall treat the left invariant normal almost contact structure on a Lie group and show that the problem can be reduced to a purely algebraic one in Lie algebras, and we shall prove that every compact Lie group of odd dimension admits a left invariant normal almost contact structure.