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Affine transformations in a differentiable manifold with Π-structure

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Introduction

Following M. Obata [4], we denote by M a manifold of even dimension $2n$ with an almost complex structure F, and by $H(x)$, $x \in M$, the homogeneous holonomy group of M with respect to a natural connection, i.e., an affine connection with respect to which F is covariant constant. $A(M)$ denotes the group of all affine transformations of M onto itself and $A_0(M)$ the connected component of the identity of $A(M)$. $QL(l, R)$ denotes the real representation of the quaternion linear group $QL(l, C)$. We assume that $H(x)$ is irreducible in R. The following theorem was proved in [4].

THEOREM A. *If n is even, $n=2l$, and $H(x)$ is not a subgroup of $QL(l, R)$, or if n is odd, then $A_0(M)$ preserves the almost complex structure. If n is even, $n=2l$, and $H(x)$ is a subgroup of $QL(l, R)$, then M has three independent almost complex structure F, G and H such that $FG=-GF=H$, $GH=-HG=F$, $HF=-FH=G$ and they are all parallel. $A(M)$ acts on the vector space spanned by F, G and H as a group of orthogonal transformations. Furthermore these orthogonal transformations belong to $SO(3)$ in the vector space.*

On the other hand, the notion of Π-structure on a differentiable manifold of any dimension m (not necessarily even) was introduced by D. C. Spencer [6]. (The name 'Π-structure' was given by G. Legrand [1].) It is one of the generalizations of the almost complex structure. Then the question arises if $A(M)$ preserves the Π-structure. An answer to this question will be given in § 2.1 as Theorem 2.

In § 1 we shall summarize briefly the known results on the Π-structure and the Π-connection. In § 2, we shall prove the main result.

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