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On local class field theory*

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The purpose of this paper is to show how the basic theorems of local class field theory can be deduced without recourse to the somewhat cumbersome computations of indices of norm class groups. Basic facts employed will be the properties of unramified (cyclic) extensions of complete fields assuming that the residue class fields with respect to the given valuation have for each positive integer exactly one extension of that degree[1].

The classical inequalities are seen to be Chevalley's famous Théorème 0[2], that is, existence of a cocycle in the second cohomology whose order equals the degree of the normal extension in question, and a lemma, implicit in the work of Kummer on explicit laws of reciprocity,[3] which states that the second cohomology group is trivial if the residue class field is algebraically closed. Throughout the paper constant use is made of the properties of Hochschild's transfer mapping[4]. The theory of the norm residue is obtained as a simple corollary to the isomorphism theorem for abelian extensions which is formulated by means of the homomorphism first used successfully by Akizuki and Nakayama[5]. Thus, the approach presented here does not make any distinction between the "characteristic unequal and equal cases" as required heretofore, if the theory of algebras as such is to be excluded.

Suppose that K/F is a normal extension of degree n with the Galois group $\mathcal{K}=\{\sigma, \tau, \rho, \cdots\}$. In the sequel the second cohomology group $H^2(\mathcal{K}, K^*)$[6] and homomorphisms related to it will be of primary importance. This cohomology group may be defined[7] as the factor group of 2-cocycles (factor sets) $f(\sigma, \tau) \in K^*$ (which form a group $Z^2(\mathcal{K}, K^*)$) satisfying the n^3 relations

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1) As to the significance of this assumption as a necessary condition for the validity of local class field theory see [**14**], [**17**] and [**19**]. The numbers in [...] refer to the bibliography at the end of the paper.

2) See [**3**, p. 142]; and [**9**, p. 341].

3) See [**8**]; and [**20**, p. 508].

4) See [**9**].

5) See [**2**], [**9**] and [**15**].

6) K^* denotes the multiplicative group of non-zero elements in K. See [**9**] for the definitions of cocycle, coboundary and cohomology group.

7) For definitions and proofs of algebraic properties see [**9**].