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Existence of derivations in graded algebras.

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In the present paper we shall discuss on the existence of derivations in the sense of C. Chevalley [1] in graded algebras. We shall give a new definition of a homomorphism of graded algebras which is a generalization of the usual one. Such a homomorphism will naturally lead us to a definition of free graded algebras as a generalization of the concept of Z-graded free algebras. The free graded algebras will play a fundamental rôle in our study.

Section 1 shows the existence of derivations of the free graded algebras. Section 2 deals with transferability between the derivation of a graded algebra and that of its homomorphic image. In the last section 3 a criterion for the existence of derivations in any graded algebras is obtained by using new binary operations which are generalizations of the usual partial differential operators.

§**1.** Throughout this paper an algebra means an algebra with a unit element 1, and a homomorphism of algebras means a ring homomorphism which maps unit upon unit. We denote by $\Gamma, \Delta, \cdots$ additive (commutative) groups and by (E, Γ) a Γ-graded algebra over any fixed (commutative or non-commutative) ring A with a unit element.

Let $E=\sum_{\gamma\in\Gamma}^{\oplus}E_\gamma$ and $F=\sum_{\delta\in\Delta}^{\oplus}F_\delta$ be decompositions of (E, Γ) and (F, Δ) into homogeneous modules respectively. Let φ_R be a homomorphism from E onto (into) F as algebras, and φ_G a homomorphism from Γ onto Δ. If $\varphi_R(E_\gamma) \subseteq F_{\varphi_G(\gamma)}$, then $\varphi=(\varphi_R, \varphi_G)$ is called a homomorphism from (E, Γ) onto (into) (F, Δ). For convenience, we write $\varphi(x)=\varphi_R(x)$ for $x\in E$, and $\varphi(\gamma)=\varphi_G(\gamma)$ for $\gamma\in\Gamma$; the kernel of φ means the kernel of φ_R.

Let (E, Γ) be a Γ-graded algebra over A. Let Δ be a homomorphic image or a factor group of Γ. Then it is easy to see that E is also a Δ-graded algebra. An element in the homogeneous module E_δ of E is called Δ-homogeneous of degree δ. A submodule M of E is said to be Δ-homogeneous if $M=\sum_{\delta\in\Delta}^{\oplus}(M\cap E_\delta)$. If a submodule M or an ideal $\mathfrak{A}$ of E is generated by Δ-homogeneous elements, then it is Δ-homogeneous by Theorem 1.3 in [1].

THEOREM 1. *Let (E, Γ) be a graded algebra over A. If Δ is a factor group of Γ, then there exists, for any Δ-homogeneous two-sided ideal $\mathfrak{A}$ of E, a homomorphism from (E, Γ) onto $(E/\mathfrak{A}, \Delta)$. Conversely, if $\varphi=(\varphi_R, \varphi_G)$ is a homomor-*