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Local theory of rings of operators I.

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The theory of rings of operators founded by F. J. Murray and J. v. Neumann [1], [2], [3], [4] was extended from the case of factors to general rings of operators by J. Dixmier [5], I. Kaplansky [6], I. E. Segal [7], and others. In particular, the notions of finiteness, and types I, II etc. of general operator algebras and of the trace of elements of these algebras were defined and investigated by these authors. The aim of this paper is to reestablish and generalize some results of these authors from a unified standpoint by introducing the notion of "local properties" of systems of elements of operator algebras.

We shall explain in § 1 what we mean by "local" and "global" properties of systems of elements of a B^*-algebra, and study mutual relations between them.

In § 2 we refer to some general theorems as preliminaries to §§ 3, 4. These are mostly known results, but we give also proofs for completeness' sake. Especially the results on "natural supporters" as named by Ti. Yen [8] after the idea of Dixmier [5], are given here for arbitrary AW^*-algebras, whereas Dixmier [5] introduced them in case of finite W^*-algebras and Ti. Yen considered them only in case of finite AW^*-algebras.

In § 3, we shall develope a "local theory" of AW^*-algebras. We shall first reestablish an important theorem of Kaplansky [6] on the equivalence between projections in AW^*-algebras as Proposition 3.5, and obtain finally a "decomposition theorem" as Proposition 3.10. The method of "localization" will turn out to be very useful in the course of this §.

Finally we shall deal with the trace in § 4. This concept was introduced by F. J. Murray - J. von Neumann [1], [2] in finite factors, and investigated further by J. Dixmier [5] in case of finite W^*-algebras, by Ti. Yen [8] and M. Goldman [9] in case of finite AW^*-algebras. We shall obtain a necessary and sufficient condition for the existence of "local trace" in finite AW^*-algebras (Proposition 4.1.) and some sufficient conditions for the existence of trace in these algebras (Theorems 4.1, 4.2, 4.3).

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