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On the fundamental conjecture of *GLC* V.

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In this paper we shall introduce the notion of regular proof-figures in *GLC* (§ 1), and prove that the end-sequence of such a proof-figure is provable without cut (Chap. I). This generalizes our result of [6].

From this result and the restriction theory in [2] follows immediately that no end-sequence of a regular proof-figure in G^1LC can contain an inconsistency of the theory of natural numbers, i.e. the logical system consisting of regular proof-figures of G^1LC and the theory of natural numbers—we shall denote this system with R^1NN for a while—is consistent. The system obtained in replacing G^1LC in the above definition of R^1NN by *GLC* will be denoted by *RNN*. The consistency of *RNN* could be also proved as an application of the result of Chapter I, but this would involve some complications. In Chapter II we prove the consistency of *RNN* by an anologous method as in Chapter I.

In this paper, we make use of the theory of ordinal diagrams, as developed in [7]. We shall show in [8] that the theory of ordinal diagrams can be formalized in *RNN*.

Chapter I. The regular proof-figure and the fundamental conjecture.

§ 1. Several concepts concerning a proof-figure of *GLC* and lemmas on ordinal diagrams.

We refer to [6], Chapter I as to the notations and the notions on *GLC* such as *t*-variables, *f*-variables, words, positive and negative, proper and inproper, degenerate and non-degenerate. We remind further that we have introduced in [5] 1.1, the notions of a formula *in a proof-figure* $\mathfrak{P}$, and of a logical symbol or a variable *in a formula A*. As these notions are of frequent use in the sequel, we shall illustrate them by an example. The same logical symbol $\forall$ may appear in a formula A as the outermost symbol and again several times. (E. g. $A=\forall\varphi\forall\psi\neg\forall\xi\neg\forall x(\xi[x]\vdash\!\dashv\varphi[x]\wedge\psi[x])$.) To distinguish these $\forall$'s, we shall designate the outermost one by ⅄, the second one by $\sharp$, the third one by $\natural$ etc. (so that $A=$ ⅄$\varphi\sharp\psi\neg\natural\xi\neg\forall x(\xi[x]\vdash\!\dashv\varphi[x]\wedge\psi[x])$ in the above example). These ⅄, $\sharp$, $\natural$,$\cdots$, *symbols considered together with the places they occupy in the formula A* are examples of symbols *in the formula*